

# Dispersed Information and CEO Incentives\*

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## Abstract

I measure the social cost of stock-based compensation schemes in a model in which the CEO learns from market prices. In my model, all agents commit a small correlated error when forming their expectations about future productivity. The equilibrium stock price thus aggregates private information with noise. I show that a stock-based compensation scheme leads the CEO to overuse the price information by a factor of three, which in turn makes the excess return and investment growth excessively volatile. I calibrate a DSGE model that embeds this mechanism, and estimate an implied welfare loss of 0.55% of permanent consumption. Surprisingly, if households were given the choice within this model of preserving the status quo or forcing the CEO to ignore all price information, they would choose the latter.

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# 1 Introduction

Executive compensation has always been a controversial topic.<sup>1</sup> In particular, the public has focused its attention on stock-based compensation as one of the primary reasons for the recent financial crisis. A commonly quoted objection is that under this form of compensation, CEOs are tempted to concentrate too much on “the markets” when making their investment decisions.

In this paper, I offer a theoretical foundation for this notion: under stock-based compensation, the CEO relies too much on market prices, which brings more noise into the real and financial side of the economy, thus causing welfare losses. In my model, all agents are *near-rational*: they make small errors when forming their expectations about the future, which renders the stock price noisy on the one hand. On the other hand, the stock price reflects the agents’ private information as well. Thus, as in a real market, the stock price is neither perfectly revealing nor pure noise but provides clouded information. The firm’s capital investment decision is delegated to the CEO, who can improve the investment allocation by inferring information from the stock price. In this setting, stock-based compensation is inefficient because it induces the CEO to commit a socially costly error when choosing capital investment. In a constrained efficient allocation that takes the agents’ near-rationality as a given, the CEO should allocate less weight toward stock price information when forming his expectation about the future. The competitive equilibrium thus suffers from a CEO that follows the financial market too closely when determining firm investment.

I first analyze the efficiency of stock-based compensation in a simplified static model. A representative firm hires a CEO to decide on its capital investment. This assumption constitutes the only possible agency conflict in the model: the CEO acts in his own self-interest, which does not necessarily coincide with household welfare. Although aligning the CEO’s payoff with that of the shareholders usually avoids this conflict, this logic turns out to be flawed in my model.

Besides capital investment, two dimensions of uncertainty influence the firm’s terminal value: internal and external uncertainty.<sup>2</sup> The CEO is precisely informed about internal

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<sup>1</sup>See Murphy (2012) for an excellent survey about the literature on executive compensation.

<sup>2</sup>I refer to information about internal (external) uncertainty as “inside (outside) information.”

uncertainty, whereas households are endowed with outside information only. Hence, the CEO is the best-informed agent in economy. However, he does not necessarily know everything insofar that some aspects are the households' exclusive knowledge. The CEO has one possibility to acquire some of this outside information: the equilibrium stock price partially reflects the households' private information when they trade in the stock market. Because of the near-rational behavior of all agents, this aggregation of outside information is clouded with noise, but it still serves as an endogenous public signal. Therefore, each agent - and in particular the CEO - can learn from the stock price.

The question is how much the CEO should learn from the stock price; that is, how much weight he should put on this signal. Intuitively, one would expect him to weight this signal by its relative precision. However, the answer is more complex in this setup: although this intuition works out for the CEO, it does not lead to a social optimum. This mismatch of private and social incentives results from the following externality: although a slight deviation from the fully rational amount of capital investment is individually inexpensive, it is considerably costlier from a social point of view. The reason for this amplification is the positive correlation between this deviation and the equilibrium stock price. The near-rational error is correlated in the cross section of agents and is therefore reflected in the stock price. Because the CEO takes the equilibrium stock price as given when he decides on capital investment, this correlation does not affect his private loss. From the households' ex-ante perspective, however, this correlation is costly because it leads to a systematic misallocation of capital.

In the constrained first-best outcome with full discretion, the social planner takes the CEO's near-rational behavior as given but can decide on his use of information. Surprisingly, the equilibrium under stock-based compensation is not even constrained efficient: the social planner can mitigate the social cost resulting from the CEO's near-rational behavior by reducing the weight attached to price information in the CEO's capital investment decision. The implied ex-ante distribution of capital investment exhibits a lower mean and variance. Therefore, stock-based compensation leads to excessively high and volatile capital investment in equilibrium compared to the constrained first-best. Importantly, these two socially costly distortions in the CEO's capital investment decision are due solely to his inefficient use of information.

Next, I consider an alternative contract in which CEO compensation is a function of

two components: (i) the firm's terminal payoff as before and (ii) a bonus that rewards the CEO when the private marginal  $q$  resulting from his investment decision is close to the average  $q$ . This contract nests both the competitive equilibrium and the constrained first-best with full discretion as special cases but also allows the social planner to limit the degree of CEO discretion by tying investment to the average  $q$ . I show the socially optimal contract always limits CEO discretion such that the stock-based component is down-weighted. The relative weight of stock-based compensation is particularly small, if the CEO's inside information is relatively unimportant compared to outside information. If, on the other hand, future productivity is primarily driven by inside information, the constrained efficient contract gives the CEO a high degree of discretion such that firm investment can effectively reflect the CEO's informational advantage.

Given the intuition built in the simplified static model, I implement the same mechanism in a dynamic setting and calibrate the model to match key macroeconomic and financial data moments. To this end, I decentralize a standard production-based asset pricing model by allowing for a cross section of privately informed households. As before, I assume the firm's investment decision is delegated to a utility-maximizing CEO who receives information about certain aspects of the firm's future productivity.

Furthermore, risk-averse households supply labor to a representative firm and invest their wealth in stocks and bonds. As a result, the firm's equilibrium stock price aggregates the households' private information and serves as an endogenous public signal that the CEO can use to improve his capital investment decision. Together with endogenously supplied labor, and a persistent productivity shock, the CEO's capital investment decision determines the firm's output.

To solve the model, I show the agents' signal extraction problem is independent of the aggregate dynamics. As a result, the agents' conditional expectations take exactly the same form as in the simple model. I solve the model numerically using perturbation methods, taking the households' average expectation about future productivity as an additional state variable. In equilibrium, all agents in the economy can extract this average expectation from the firm's stock price to update their beliefs about future productivity. Using this methodological framework, I can extend a standard DSGE model with the realistic notion that prices play an informative role. In this adjusted dynamic model, I can quantify the efficiency loss accruing from stock-based compensation.

After calibrating the model, I compare the competitive equilibrium under stock-based compensation to the constrained first-best. This analysis yields three main results: (i) the CEO relies too much on price information when forming his expectation about the future, (ii) this distortion leads to a welfare loss of about 0.55% of life-time consumption, and (iii) the excess stock return and investment growth are excessively volatile. Furthermore, I compare these two scenarios to an economy in which the CEO is forced to ignore information revealed by the stock price. Relative to the constrained first-best, this alternative setting leads to a welfare loss of about 0.46% of lifetime consumption. Interestingly, this loss is smaller than that associated with the competitive equilibrium. As a result, in my preferred calibration, the households are better off relative to the competitive equilibrium if the CEO ignores information transmitted through the stock price.

The idea that efficient stock markets aggregate private and public information into stock prices is a central topic in financial economics and can be traced back to Hayek (1945). The extent to which prevailing prices are informative about the future value of a firm is important for both traders and real decision makers, such as firm managers, central bankers, or politicians. The more information these agents can extract from stock prices, the more they can improve on their economic decisions, such as trading, corporate investment, and policy interventions. This notion that real decision makers appear to learn information from prices has received recent empirical support from Luo (2005), Chen, Goldstein, and Jiang (2007), Bakke and Whited (2010), Edmans, Goldstein, and Jiang (2012), and Foucalt and Fresard (2013).

This key insight of informative prices led to the large literature on noisy rational expectations equilibria following Hellwig (1980) and Diamond and Verrecchia (1981). In these models exogenous noise trading clouds the informational content of stock prices. In this paper, I follow Hassan and Mertens (2011) and Hassan and Mertens (2014b), by introducing near-rational errors instead of noise traders. This approach allows me to make inferences about investor welfare. The notion of near-rationality puts discipline on the amount of noise in equilibrium asset prices, which is consistent with the idea that losses to individual traders causing this noise must be economically small.<sup>3</sup>

A recent literature models an informational feedback effect from the financial market

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<sup>3</sup>See also Akerlof and Yellen (1985), Cochrane (1989), and Chetty (2012) for applications of near-rationality.

to firm decisions. For instance, in Subrahmanyam and Titman (2001), Subrahmanyam and Titman (2013), Goldstein, Ozdenoren, and Yuan (2013), and Goldstein and Yang (2014), informative signals originating from the financial market influence the firm’s investment decision.<sup>4</sup> Relative to this literature, my contribution lies in the analysis of socially optimal managerial incentives and the quantification in a dynamic asset pricing model.

A number of papers has studied the relationship between stock-based managerial contracts and the efficiency of corporate investment decisions, including Diamond and Verrecchia (1982), Stein (1989), Benmelech, Kandel, and Veronesi (2010), Strobl (2014), and Peng and Roell (2014). A recent literature in dynamic corporate finance quantifies the impact of different agency conflicts found in this literature. See, for example, Morellec (2004), Glover and Levine (2014a), and Glover and Levine (2014b).<sup>5</sup> Thanks to recent developments in numerical methods, I can contribute to this literature by solving a standard DSGE model with the agency friction of delegated capital investment. To this end, I make use of perturbation methods as in Judd and Guu (2001), Mertens (2011), Hassan and Mertens (2014a), Hassan and Mertens (2014b), Tille and van Wincoop (2014a), and Tille and van Wincoop (2014b).

The remainder of the paper is organized as follows: section 2 sets up and solves a simplified static model with delegated investment and information aggregation by prices. The key inefficiencies are highlighted in section 3. Section 4 embeds the static setup into a dynamic production-based asset pricing model.

## 2 A Simple Model

A continuum of risk-neutral agents is indexed by  $i \in [0, 1]$ , and one firm issues claims to its profits.<sup>6</sup> A CEO who decides on the level of capital accumulation runs the firm. The terminal payoff depends on the CEO’s investment decision and realized productivity. Two time periods exist: in  $t = 1$ , households trade shares in a financial market and the CEO

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<sup>4</sup>In Angeletos, Lorenzoni, and Pavan (2010), the feedback effect is “reversed” as information flows from the real sector to the financial market. Moreover, in Bond and Goldstein (2014), the government is able to learn information from the financial market.

<sup>5</sup>See Strebulaev and Whited (2011) for an excellent survey of the literature on dynamic models in corporate finance.

<sup>6</sup>I make the assumption of a single, representative firm solely to keep the notation simple. In Appendix C.1, I show the main conclusions from this section carry over to a setup with a continuum of firms.

makes an investment decision. Both the households and the CEO base their conditional expectation of the productivity shocks on the equilibrium stock price and private signals. In the last period, uncertainty is resolved: the terminal payoff is determined and all agents get paid.

## 2.1 Firm Decision

The representative firm has access to the following linear production technology:

$$Y = e^{(\theta_a + \theta_f)} K, \quad (1)$$

where  $K$  denotes capital investment and  $\{\theta_a, \theta_f\}$  are two independent productivity shocks that are distributed as

$$\theta_a \sim N(0, \sigma_a^2) \text{ and } \theta_f \sim N(0, \sigma_f^2). \quad (2)$$

The two shocks  $\theta_a$  and  $\theta_f$  represent two independent sources of uncertainty that affect the terminal payoff. Information about  $\theta_a$  is dispersed among all households in the economy in such a way that they receive a private signal about its realized value. As a consequence, the equilibrium stock price aggregates and reflects information about this shock such that each agent is able to extract information from the stock price. Information about  $\theta_f$  is restricted to the CEO; that is, households only know the prior distribution. As a result, the equilibrium stock price does not reveal any information about  $\theta_f$ .

This information structure captures the idea that on the one hand, the CEO has an informational advantage vis-a-vis the households regarding certain aspects affecting the firm's profitability. This pool of information reflects internal information, such as corporate strategies, planned mergers, product development, and so on. This dimension of uncertainty is reflected in  $\theta_f$ . On the other hand, he is able to learn about external factors from the firm's stock price that reveals information about  $\theta_a$ . This additional pool of information can include the state of the economy, the position of competitors, consumer demand, and so on. Therefore, the CEO knows more about future productivity than anybody, but not everybody, in the market.

In  $t = 1$ , the CEO chooses capital investment  $K$  to maximize his expected terminal

wealth  $W_{\text{CEO}}$  specified below:

$$\max_K \mathcal{E}_{\text{CEO}} [W_{\text{CEO}}]. \quad (3)$$

I assume the CEO starts with zero initial wealth such that his only source of income is the compensation paid by the firm. In the benchmark economy, this compensation is proportional to the firm's terminal payoff, that is, firm value net of investment cost. The latter is increasing in the CEO's capital investment decision, and for simplicity, I use the following quadratic specification for the capital adjustment cost:

$$C \equiv \mathcal{C}(K) = \frac{1}{2}K^2. \quad (4)$$

Then, the CEO's compensation is given by

$$W_{\text{CEO}} = \omega_{\text{CEO}} (Y - C), \quad (5)$$

where  $\omega_{\text{CEO}} \in [0, 1)$  represents the fixed share of the firm's terminal payoff given to the CEO.<sup>7</sup> The remainder is then paid out to the households such that the firm's terminal payoff is given by

$$D = Y - C - W_{\text{CEO}} = (1 - \omega_{\text{CEO}}) (Y - C). \quad (6)$$

As a result, the CEO chooses  $K$  to maximize:

$$\max_K \mathcal{E}_{\text{CEO}} \left[ \omega_{\text{CEO}} \left( e^{(\theta_a + \theta_f)} K - \frac{1}{2}K^2 \right) \right]. \quad (7)$$

The CEO's expectation is conditional on his private signal about  $\theta_f$  as well as on the equilibrium stock price.<sup>8</sup>

## 2.2 Information Structure

All agents know the prior distribution for both productivity shocks and observe the asset price  $P$ . Households receive a private signal about  $\theta_a$ :

$$x_i = \theta_a + \nu_i \quad \forall i \in [0, 1], \quad (8)$$

<sup>7</sup>Clearly, adding a fixed component to the CEO's terminal compensation does not alter his decisions in equilibrium.

<sup>8</sup>For convenience, I assume the CEO's outside option is equal to zero; that is, his expected compensation needs to be weakly positive to satisfy the participation constraint. Moreover, I abstract from any effort costs to isolate the CEO's expectation formation.



where  $\nu_i$  is i.i.d. across households and normally distributed with mean zero and standard deviation  $\sigma_x$ . The CEO receives a private signal about  $\theta_f$ , and for simplicity, I assume this signal to be perfectly informative so that the CEO observes  $\theta_f$  at  $t = 1$  without noise.

### 2.3 Households

At  $t = 1$ , households submit price-dependent orders to trade claims to the terminal payoff  $D$ . They can buy or sell shares  $\omega_i$  inside the limits  $[\underline{\omega}, \bar{\omega}]$  with  $\underline{\omega} < 1 < \bar{\omega}$ . These position limits are necessary to keep optimal portfolio shares finite and can be interpreted as borrowing or short-selling constraints. In general, the specific values for these limits do not matter and it is sufficient to rule out unlimited positions. Because households are risk-neutral, they choose the portfolio share  $\omega_i$  that maximizes their expected terminal wealth conditional on  $t = 1$  information.<sup>9</sup>

I assume every household is endowed with one share in the beginning; that is,  $W_0 = P$ . Then, households choose holdings in the risky asset such that

$$\max_{\omega_i \in [\underline{\omega}, \bar{\omega}]} \mathcal{E}_i[W_i], \quad (9)$$

with  $W_i = W_0 \left( \omega_i \frac{D}{P} + (1 - \omega_i) \right)$ . Therefore, households receive the portfolio return weighted by their initial wealth. Because of linear preferences and zero discounting, the risk-free rate is equal to zero and each household purchases either  $\underline{\omega}$  or  $\bar{\omega}$  shares.

I assume all agents commit a correlated *near-rational* error as in Hassan and Mertens (2014b) when forming their expectation about  $\theta_a$ .<sup>10</sup> As a result, their posterior probability density function is shifted by  $\varepsilon + \varepsilon_i$ , where  $i \in [0, 1]$ . I assume the correlation to be positive so that  $\varepsilon \sim N(0, \sigma_\varepsilon^2)$  is the common component. The idiosyncratic part is distributed as  $\varepsilon_i \sim N(0, \hat{\mu}\sigma_\varepsilon^2)$ , where  $\hat{\mu}$  calibrates the size of the correlation.<sup>11</sup> Thus, the near-rational expectation is given as

$$\mathcal{E}_i[\theta_a] = E_i[\theta_a] + \varepsilon, \quad (10)$$

where  $E_i[\theta_a] = E[\theta_a | x_i, P]$  denotes the rational Bayesian expectation. Importantly, even

<sup>9</sup>I generalize the assumption of risk-neutrality in the quantitative model discussed in section 4.

<sup>10</sup>The assumption that the near-rational error is associated with the expectation about  $\theta_a$  is without loss of generality: equivalently, it can be associated with the expectation about  $\theta_f$  or  $\theta_a + \theta_f$ .

<sup>11</sup>For simplicity, I set  $\hat{\mu} = 0$  such that the common error is perfectly correlated.

though near-rational agents commit a mistake in their expectation, they fully understand the structure of the economy, the equilibrium mapping of information into the stock price, and all higher moments of  $\theta_a$ .

Here,  $\sigma_\varepsilon$  governs the magnitude of the near-rational error, that is, the distance between the optimal and near-rational expectation. As  $\sigma_\varepsilon \rightarrow 0$ , all agents become fully rational.

Market clearing requires that the aggregate demand equals the fixed supply of the asset. I normalize the supply of stocks to 1; thus,

$$\int_0^1 \omega_i di = 1. \tag{11}$$

The market-clearing condition then determines the equilibrium stock price  $P$ .

## 2.4 Equilibrium Definition

**Definition 1** *An equilibrium consists of a price function,  $P(\theta_a, \varepsilon) : \mathbb{R}^2 \rightarrow \mathbb{R}$ , an investment policy for the CEO,  $K(\theta_f, P, \varepsilon) : \mathbb{R}^3 \rightarrow \mathbb{R}$ , and a trading strategy for households,  $\omega(x_i, P, \varepsilon) : \mathbb{R}^3 \rightarrow [\underline{\omega}, \bar{\omega}]$  such that:*

- (a) *the CEO and each household maximize expected wealth and*
- (b) *the stock market clears.*

## 2.5 Model Solution

The necessary steps to solve the simple model are the following: first, I solve the signal-extraction problem for each household and the CEO, respectively. Second, I derive the optimal decisions on trading and capital investment given the equilibrium expectations.

### Equilibrium Expectations

As argued above, households optimally demand  $\bar{\omega}$  if their conditional expectation of the payoff exceeds the price, and  $\underline{\omega}$  otherwise. Hence, the market-clearing condition implies the portfolio share  $\underline{\omega}$  that the “pessimists” demand plus the quantity  $\bar{\omega}$  that the “optimists” demand equals the unit supply:<sup>12</sup>

$$\Pr(\mathcal{E}_i[D] \leq P) \underline{\omega} + \Pr(\mathcal{E}_i[D] > P) \bar{\omega} = 1, \tag{12}$$

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<sup>12</sup>Here, I rule out “insider trading” by the CEO; that is, he is not allowed to implicitly alter the number of shares through trading in the stock market.

where the probabilities are cumulative distribution functions of  $\nu_i$ , that is, conditional on aggregate shocks.

I solve the signal-extraction problem by first guessing the equilibrium stock price is a monotonic function of the average expectation of  $\theta_a$ .

**Conjecture 1** *Learning from the stock price  $P$  is equivalent to learning from the average expectation  $\hat{q} \equiv \int_0^1 \mathcal{E}_i[\theta_a] di$ , such that  $P$  and  $\hat{q}$  span the same  $\sigma$ -algebra.*

Given Conjecture 1, both variables contain the same amount of information about the productivity shock  $\theta_a$ . Mathematically, using  $\hat{q}$  instead of  $P$  simplifies the analysis significantly because it allows me to use the standard projection theorem for normally distributed random variables.<sup>13</sup> Therefore, I solve the model using this guess and show the resulting equilibrium stock price is indeed informationally equivalent to the average expectation  $\hat{q}$ . In the following, I refer to learning from  $\hat{q}$  as *learning from the stock price*.

Consequently, the conditional (rational) expectation of  $\theta_a$  for household  $i$  is a linear function of prior information, the private signal  $x_i$ , and price information captured through  $\hat{q}$ :

$$E_i[\theta_a] = \alpha_1 x_i + \alpha_2 \hat{q}, \quad (13)$$

where  $\{\alpha_1, \alpha_2\}$  are Bayesian weights determined in equilibrium.

Plugging in the definition of  $\hat{q}$  from Conjecture 1 and integrating over all households, implies

$$\hat{q} = \frac{\alpha_1}{1 - \alpha_2} \theta_a + \frac{1}{1 - \alpha_2} \varepsilon. \quad (14)$$

Thus, the transformed stock price (or equivalently, the households' average expectation)  $\hat{q}$  is informative about  $\theta_a$ . The aggregation, however, is not perfect because it is clouded by the near-rational error  $\varepsilon$ . The response of  $\hat{q}$  to  $\varepsilon$  depends on the weight  $\alpha_2$  that all individual households attach to price information. Because  $\alpha_2 \in [0, 1)$ , it follows that the near-rational error is amplified in equilibrium. This amplification is particularly strong if each household relies heavily on the price signal.

<sup>13</sup>Note that both productivity shocks  $\{\theta_a, \theta_f\}$ , private signals  $\{x_i\}$ , and near-rational errors  $\varepsilon$  are jointly normally distributed. The guess that  $\hat{q}$  is also normal is verified in equation (14).

The expression for  $\hat{q}$  above implies each household and the CEO can extract information about  $\theta_a$  from the unbiased signal  $s_p$ :

$$s_p \equiv \frac{1 - \alpha_2}{\alpha_1} \hat{q} = \theta_a + \frac{1}{\alpha_1} \varepsilon. \quad (15)$$

This signal's precision,  $\sigma_p^{-2} \equiv \alpha_1^2 \sigma_\varepsilon^{-2}$ , measures price efficiency in the economy because it determines how much information the (transformed) stock price reveals about  $\theta_a$ . It depends positively on two factors: the degree of rationality ( $\sigma_\varepsilon^{-2}$ ) and the weight the households attach to their private signal ( $\alpha_1$ ). This expression shows that the stock price becomes perfectly revealing in the limit as  $\sigma_\varepsilon \rightarrow 0$ , that is, as all agents become perfectly rational.

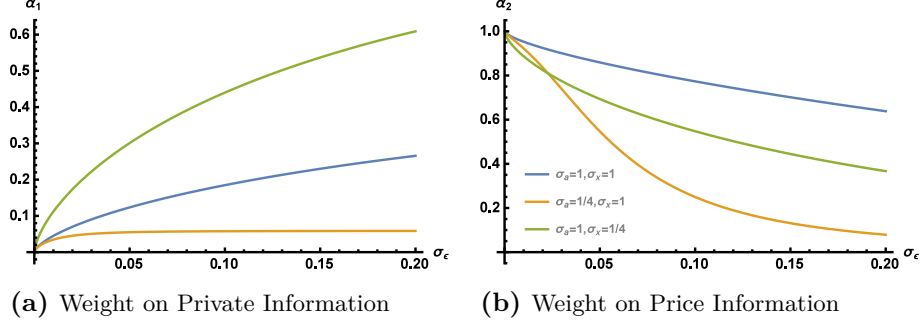
From the projection theorem for normal random variables, it follows that the Bayesian weights in the conditional expectation (13) are given by

$$\alpha_1 = \frac{\sigma_x^{-2}}{\sigma_a^{-2} + \sigma_x^{-2} + \sigma_p^{-2}} \quad (16)$$

$$\alpha_2 = \frac{\sigma_p^{-2}}{\sigma_x^{-2} + \sigma_p^{-2}}. \quad (17)$$

Because price efficiency  $\sigma_p^{-2}$  also depends on  $\alpha_1$ , the system above constitutes an implicit solution for the coefficients in  $E_i[\theta_a]$ .<sup>14</sup> Intuitively, the optimal weights depend on the relative precisions of private and price information, that is,  $\sigma_x^{-2}$  and  $\sigma_p^{-2}$ . Figure 1 shows a plot of  $\alpha_1$  and  $\alpha_2$  for different parameter values. Panel (a) plots the weight attached to private information  $\alpha_1$  against  $\sigma_\varepsilon$ , the standard deviation of the near-rational error for three different combinations of  $\sigma_a$  and  $\sigma_x$ . For all cases,  $\alpha_1$  increases as  $\varepsilon$  becomes more volatile, because the private signal becomes relatively more attractive in this case. The blue line corresponds to the case in which  $\sigma_a = \sigma_x = 1$ . In this scenario, households allocate a weight of about 10% – 20% to their private signal. As their private signal becomes more precise (green line), this weight increases up to 60%, while it decreases to below 10% if their prior information becomes more precise (orange line). Panel (b) conducts the same exercise for the weight on price information  $\alpha_2$ . It can be observed that this weight decreases uniformly in  $\sigma_\varepsilon$ . Moreover, the weight is highest if both  $\sigma_a$  and  $\sigma_x$  are relatively high (blue line). As either of these standard deviations decreases, the households optimally decrease their weight on price information.

<sup>14</sup>An explicit solution for  $\alpha_1$  is provided in Appendix A.2.



**Figure 1:** Bayesian weights  $\alpha_1$  and  $\alpha_2$  in (16) and (17) for different parameter values. Blue line:  $\sigma_a = \sigma_x = 1$ ; Orange line:  $\sigma_a = 1/4, \sigma_x = 1$ ; Green line:  $\sigma_a = 1, \sigma_x = 1/4$ .

### Optimal CEO Behavior

At  $t = 1$ , the CEO maximizes his expected terminal wealth defined in (5) by choosing  $K$  such that optimal capital investment is given by

$$K = \mathcal{E}_{\text{CEO}} [\exp(\theta_a + \theta_f)], \quad (18)$$

where the CEO's near-rational expectation is conditional on private information about  $\theta_f$  as well as on price information.

Taking logs on both sides of (18) and using the fact that the CEO's conditional expectation of  $\theta_a$  is linear in his signals (due to the normality of all signals) gives

$$k = \beta_0 + \beta_1 \hat{q} + \varepsilon + \theta_f, \quad (19)$$

where the expressions for  $\beta_0$  and  $\beta_1$  follow from the projection theorem:

$$\beta_0 = \frac{1}{2} (\sigma_a^{-2} + \sigma_p^{-2})^{-1} \quad (20)$$

$$\beta_1 = \frac{(1 - \alpha_2)}{\alpha_1} \sigma_p^{-2} (\sigma_a^{-2} + \sigma_p^{-2})^{-1}. \quad (21)$$

Intuitively, the CEO invests more if he is optimistic about the realization of either productivity shock. Although his perception of  $\theta_f$  is accurate, two factors influence his belief about  $\theta_a$ : price information  $\hat{q}$  and the correlated error  $\varepsilon$ .

## Optimal Trading Behavior

Because the terminal payoff is given by  $D = (1 - \omega_{\text{CEO}})(Y - C)$ , the expected payoff for household  $i$  is given by

$$\mathcal{E}_i[D] = (1 - \omega_{\text{CEO}}) \left( \exp \left( \mathcal{E}_i[y] + \frac{1}{2} V_i[y] \right) - \frac{1}{2} \exp (2\mathcal{E}_i[k] + 2V_i[k]) \right), \quad (22)$$

where the expected log firm value is given by  $\mathcal{E}_i[y] = \mathcal{E}_i[\theta_a] + \mathcal{E}_i[k]$  and the expected log capital investment by  $\mathcal{E}_i[k] = \beta_0 + \beta_1 \hat{q}$ .<sup>15</sup>

Given the households' optimal trading behavior and the CEO's optimal investment decision, I can now use the market-clearing condition (11) to solve for the equilibrium stock price.

**Lemma 1** *The equilibrium stock price is given by*

$$P = \frac{\exp(\kappa_0 + (1 + \beta_1)\hat{q}) - \frac{1}{2} \exp(\kappa_1 + 2\beta_1\hat{q})}{1 - \omega_{\text{CEO}}}, \quad (23)$$

where  $\{\kappa_0, \kappa_1\}$  are constants defined in Appendix A.1.1.

**Proof:** See Appendix A.1.1.

In the Appendix, I derive a sufficient condition under which  $P$  is monotonically increasing in the average expectation of  $\theta_a$ , that is,  $\hat{q}$ .

**Condition 1** *The maximum number of shares that can be purchased,  $\bar{\omega}$ , is larger than some threshold value  $\omega^*$  defined in Appendix A.1.1.*

Under this condition, Conjecture 1 is verified such that all agents in the economy can invert the equilibrium stock price to solve for  $\hat{q}$ , which in turn serves as an informative signal about the productivity shock  $\theta_a$ .

<sup>15</sup>The conditional variances  $V_i[y]$  and  $V_i[k]$  are constant across households. The expression for both terms is given in Appendix A.1.1. Also note that I have used  $\mathcal{E}_i[\varepsilon] = 0$  such that near-rational agents are not able to predict  $\varepsilon$ .

### 3 Inefficiencies in the Simple Model

#### 3.1 Social Welfare

For the normative analysis, I first define the social welfare function as the unconditional expected utility of an arbitrary household at  $t = 0$ .<sup>16</sup>

$$SWF \equiv \int_0^1 E_0[W_i] di \quad (24)$$

such that the households' ex-ante expected utility is proportional to the (unconditionally) expected firm profits,  $Y - C$ .<sup>17</sup>

Plugging in the expressions for the payoff  $Y$  and the investment cost  $C$  in (24) leads to the following expression:

**Lemma 2** *Social welfare can be written as*

$$\begin{aligned} SWF &= \exp\left(E_0[k] + \frac{1}{2}(\sigma_a^2 + \sigma_f^2 + V_0[k]) + Cov(\theta_a + \theta_f, k)\right) \\ &\quad - \frac{1}{2} \exp(2E_0[k] + 2V_0[k]). \end{aligned} \quad (25)$$

**Proof:** See Appendix A.1.2.

Therefore, social welfare depends on the CEO's capital investment decisions in three ways: (i) the expected level of investment, (ii) the variance of investment, and (iii) the covariance between the productivity shocks and investment.

In this environment, a social planner could clearly achieve a Pareto-superior solution by rendering all agents perfectly rational, that is, by setting  $\varepsilon = 0$  with certainty. As a result, the equilibrium stock price would perfectly reveal the true value of the productivity shock to all agents so that all asymmetric information vanishes as emphasized in Grossman (1976).

**Proposition 1** *In the first-best, with  $\sigma_\varepsilon = 0$ , the stock price perfectly reveals  $\theta_a$ . Optimal*

<sup>16</sup>This utilitarian social welfare function corresponds to the conventional definition of "real efficiency" as, e.g., in Goldstein, Ozdenoren, and Yuan (2013) and Goldstein and Yang (2014).

<sup>17</sup>Note that because households are near-rational, expected utility under the fully rational measure is almost the same as expected utility under the near-rational measure. So I do not have to worry about "respecting" agents' errors as in Brunnermeier, Simsek, and Xiong (2014).

capital investment is given by  $K_{fb} = \exp(\theta_a + \theta_f)$  and social welfare by:

$$SWF_{fb} = \frac{1}{2} \exp\left(2\sigma_a^2 + 2\sigma_f^2\right). \quad (26)$$

**Proof:** See Appendix A.1.3.

Therefore, with perfect information, social welfare solely depends on the variance of the productivity shocks. As a result, linking the CEO's compensation to the firm's terminal payoff is efficient in this setup, because his objective function lines up perfectly with household welfare.

**Corollary 1** *In the first-best, stock-based compensation is efficient.*

**Proof:** See Appendix A.1.4.

This corollary confirms the standard notion of stock-based compensation. In the absence of asymmetric information, linking CEO compensation to the stock payoff is efficient. Therefore, I highlight *informational* inefficiencies as a result of (near-rational) managerial discretion about capital investment.

### 3.2 Near-Rationality

The discussion above emphasizes the importance of agents' near-rational errors to generate any inefficiencies in the benchmark setup. As in Hassan and Mertens (2014b) the key idea behind this concept is that the *individual* cost for each agent to commit the near-rational error is negligibly small, whereas in the *aggregate*, it leads to a sizeable welfare loss.

Note that for an individual household, ex-ante expected utility is proportional to  $E_0[Y - C]$ , that is, the unconditional expectation of firm profits. Then, it follows immediately that the *individual* welfare cost of committing the near-rational error is equal to zero. This strong result is slightly altered in the presence of risk aversion, which is a feature of the quantitative model in section 4. In this setup, the intensity of the near-rational error ( $\sigma_\varepsilon$ ) is chosen such that the individual cost is below a given threshold. In other words, near-rationality does not affect the *level* of wealth (in expectation) for an individual agent.

Interestingly, in the *aggregate*, near-rational behavior has an effect on social welfare.



As shown in Hassan and Mertens (2014b), the households' near-rational errors are amplified in equilibrium through their trading in the financial market. This amplification can be seen in (14), where  $\varepsilon$  is multiplied by  $\frac{1}{1-\alpha_2} > 1$ . The weight  $\alpha_2$ , the households attach to price information thus leads to a propagation of noise in the average expectation  $\hat{q}$  (or alternatively in the stock price). This propagation in turn crowds out information about  $\theta_a$  from the equilibrium stock price, which in turn decreases investment efficiency and thus welfare.

In the setting considered in this paper, a second externality arises, which leads to the CEO's near-rational behavior. The following lemma shows the individual welfare loss for a rational CEO of using the near-rational expectation instead of the fully rational expectation when choosing capital investment.

**Lemma 3** *Let  $U_{CEO}^R$  and  $U_{CEO}^{NR}$  denote the CEO's payoff under full rationality and near-rationality, respectively. Then the ratio in expected utilities for the CEO is given by*

$$\frac{E_1 [U_{CEO}^{NR}]}{E_1 [U_{CEO}^R]} = 2 \exp\left(\frac{1}{2}\sigma_\varepsilon^2\right) - \exp\left(2\sigma_\varepsilon^2\right), \quad (27)$$

where  $\sigma_\varepsilon^2$  denotes the variance of the near-rational error  $\varepsilon$ , and  $E_1[\cdot]$  is the rational expectation conditional on information available to the CEO at  $t = 1$ .

**Proof:** See Appendix A.1.5.

Therefore, the CEO's individual cost of making a near-rational error depends only on the standard deviation of  $\varepsilon$ . As  $\sigma_\varepsilon$  converges to zero, the ratio in (27) converges to one such that the individual loss converges to zero. Moreover, it is straightforward to show that the ratio  $\frac{E_1 [U_{CEO}^{NR}]}{E_1 [U_{CEO}^R]}$  can be approximated (for  $\sigma_\varepsilon$  small) as  $1 - \sigma_\varepsilon^2$ .<sup>18</sup> As a result, the CEO's individual utility loss from committing the near-rational error is in the order of  $\sigma_\varepsilon^2$ .

The implied social costs of the CEO's near-rational investment decision is specified next.

**Lemma 4** *Let  $SWF^R$  and  $SWF^{NR}$  denote social welfare implied by capital investment under full rationality and near-rationality, respectively. Then the ratio in social welfare is given by*

$$\frac{SWF^{NR}}{SWF^R} = 2 \exp\left(\frac{1}{2}\left(1 + \frac{2\beta_1}{1-\alpha_2}\right)\sigma_\varepsilon^2\right) - \exp\left(2\left(1 + \frac{2\beta_1}{1-\alpha_2}\right)\sigma_\varepsilon^2\right), \quad (28)$$

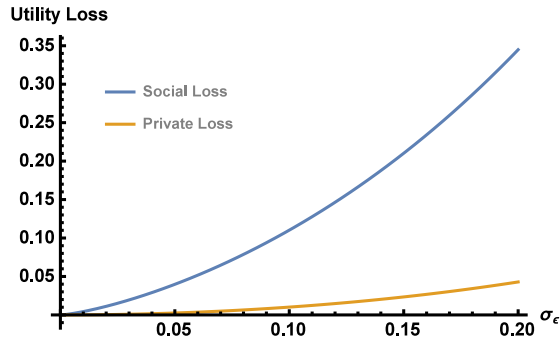
<sup>18</sup>This expression follows directly from the first-order approximation:  $e^x \approx 1 + x$ .

where  $\sigma_\varepsilon^2$  denotes the variance of the near-rational error, and  $\{\alpha_2, \beta_1\}$  corresponds to the Bayesian weight attached to price information for an individual household and the CEO, respectively.

**Proof:** See Appendix A.1.6.

As a result, the social loss associated with the CEO's near-rational behavior includes an additional term  $(\frac{2\beta_1}{1-\alpha_2})$  originating from the fact that the CEO's near-rational error is correlated with the equilibrium stock price. Again, approximating the ratio  $\frac{SWF^{NR}}{SWFR}$  leads to  $\frac{SWF^{NR}}{SWFR} \approx 1 - \left(1 + \frac{2\beta_1}{1-\alpha_2}\right) \sigma_\varepsilon^2$ . As a result, the externality can be approximated as  $\frac{2\beta_1}{1-\alpha_2} \sigma_\varepsilon^2$ , such that the difference between the individual and social loss is large if (i) the CEO relies heavily on price information or (ii) each individual household relies heavily on price information.

Figure 2 plots the CEO's private utility loss (orange line) resulting from his near-rational behavior against the social loss (blue line), that is, the percentage decrease in social welfare due to the CEO's near-rationality. Note that the private loss is considerably smaller than the social loss. As an example, consider a value of  $\sigma_\varepsilon = 0.2$ : the private loss (orange line) amounts to a rather small value of 5%, whereas the social loss (blue line) equals almost 35%. This outcome again emphasizes the notion of near-rationality: because each individual does not exert the effort to avoid small welfare losses, they pile up to a considerable social cost.



**Figure 2:** CEO's private welfare loss vs. social loss resulting from near-rational investment behavior. Parameters:  $\sigma_a = \sigma_x = \sigma_f = 1$ .

Hence, it becomes obvious that stock-based compensation is not an adequate means to prevent the CEO from making socially costly investment decisions.

In the following, I consider two investment strategies that reduce this social cost. In

the first scenario, I take the CEO's compensation structure as given and derive the *constrained* first-best investment policy, given near-rational behavior. In the second scenario, I consider a different compensation package that limits CEO discretion.

### 3.3 Constrained First-Best: Full Discretion

As shown before, the private cost for each household and the CEO of committing the near-rational error is considerably smaller than the implied social cost. In this section, I take all agents' near-rationality as given and analyze the (constrained) first-best where information remains decentralized and the households follow their equilibrium (trading) policies.<sup>19</sup> Consequently, this analysis serves two main purposes: (i) it highlights inefficiencies in the CEO's equilibrium investment decision conditional on near-rationality and (ii) it provides guidance to construct more efficient corporate governance schemes.

In the following, I characterize the constrained efficient allocation in more detail.

**Definition 2** *A feasible allocation is a collection of portfolio choices  $\omega_i$ , one for each household, and an investment decision for the CEO, that jointly satisfy the following constraints:*

(a) *Resource feasibility:*

$$\int_0^1 W_i di = Y - \frac{1}{2}K^2 - W_{CEO}, \quad (29)$$

where  $W_i$  denotes terminal consumption of each household  $i \in [0, 1]$ .

(b) *Informational feasibility:* For each household  $i$ ,  $\omega_i$  is contingent on the private signal  $x_i$  and the stock price  $P$ , for the CEO,  $K$  is contingent on  $P$  and  $\theta_f$ .

(c) *Near-rational feasibility:* Each household  $i$  and the CEO commit a near-rational error when forming their expectations about  $\theta_a$ .

**Definition 3** *A constrained efficient investment allocation is a feasible allocation that is not Pareto-dominated by any other feasible allocation, taking as given the utility-maximizing portfolio choices for each household.*

More specifically, I keep the information structure as before and allow the CEO to extract information from the stock price to predict the productivity shock  $\theta_a$ . I restrict the

<sup>19</sup>The concept of constrained efficiency is also used in Angeletos and Pavan (2007), Angeletos and Pavan (2009), and Angeletos, Lorenzoni, and Pavan (2010).

discussion to (near-rational) log-linear investment rules such that  $k = b_0 + b_1\hat{q} + b_2\theta_f + \varepsilon$ . Then the social planner chooses  $\{b_0, b_1, b_2\}$  to maximize social welfare in (24). The solution is described in the following proposition.

**Proposition 2** *The investment policy that maximizes social welfare is given by  $k = b_0 + b_1\hat{q} + b_2\theta_f + \varepsilon$ , where*

$$b_0 = \left( \frac{1}{2} - \left( 1 + \frac{3}{2}\alpha_1 \right) \alpha_1 \right) \left( \sigma_a^{-2} + \sigma_p^{-2} \right)^{-1} \quad (30)$$

$$b_1 = \frac{(1 - \alpha_2)}{\alpha_1} \left( \sigma_p^{-2} - \alpha_1 \sigma_a^{-2} \right) \left( \sigma_a^{-2} + \sigma_p^{-2} \right)^{-1} \quad (31)$$

$$b_2 = 1. \quad (32)$$

**Proof:** See Appendix A.1.7.

First, note that the social planner takes the households' trading behavior as given. As a result, the households' weights on private and price information ( $\alpha_1$  and  $\alpha_2$ ) are still given by (16) and (17), respectively. It then follows that the choice of  $\{b_0, b_1, b_2\}$  does not affect price informativeness  $\sigma_p^{-2}$ .

**Lemma 5** *Price informativeness in the constrained efficient allocation is equal to that in the competitive equilibrium.*

**Proof:** See Appendix A.1.8.

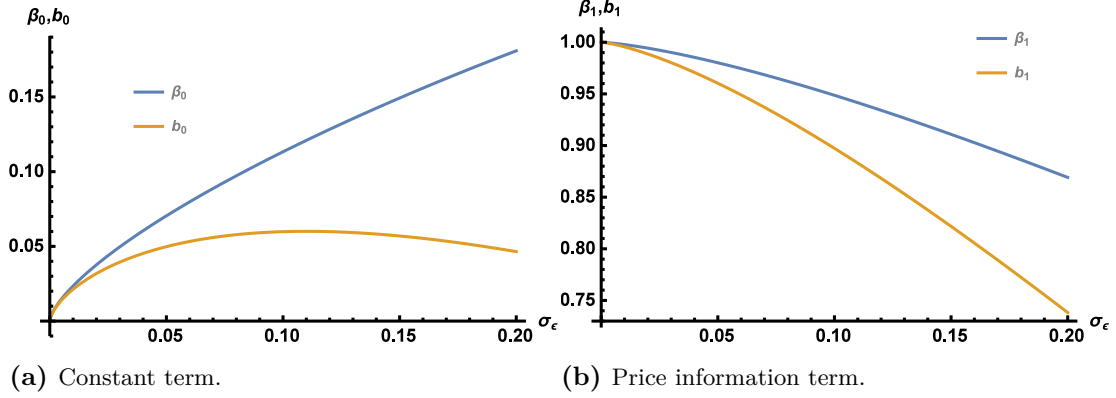
Proposition 2 states that the CEO efficiently<sup>20</sup> uses his knowledge about  $\theta_f$  in equilibrium as  $b_2 = 1$  in (32). However, (30) and (31) show the CEO allocates inefficiently high values to  $\beta_0$  and  $\beta_1$  in equilibrium. This allocation implies the CEO relies too heavily on the price signal when forming his expectation about  $\theta_a$ . Figure 3 compares  $\beta_0$  and  $\beta_1$  to their constrained efficient counterparts, and the following proposition formalizes the observations from the example in the plot.

**Proposition 3** *Relative to the constrained efficient investment rule, the CEO efficiently uses knowledge about  $\theta_f$  but chooses inefficiently high values for  $\beta_0$  and  $\beta_1$ , namely,  $\beta_0 \geq b_0$ , and  $\beta_1 \geq b_1$ .*

**Proof:** See Appendix A.1.9.

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<sup>20</sup>Note that whenever I use the term *efficient*, I refer to efficiency according to Definition 3, namely, constrained efficiency.



**Figure 3:** The CEO's weights in the capital investment policy: benchmark equilibrium vs. constrained efficient allocation. Parameters:  $\sigma_a = \sigma_f = \sigma_x = 1$ .

This result is surprising because the CEO's terminal payoff is proportional to that of an average agent. Thus, he maximizes the same objective function as the social planner, just under a different information set.<sup>21</sup>

Intuitively, the CEO chooses the weights in his investment policy that give him the most precise estimate of the productivity shocks given the information set at  $t = 1$ . For households, however, the relevant welfare measure is their unconditional expected utility at  $t = 0$ .

The basic reason for the distortion in the equilibrium is the CEO's exposure to the near-rational error  $\varepsilon$ . Because of this exposure, the *Bayesian* weights in (20) and (21) are not efficient, because they ignore the positive correlation between price information ( $\hat{q}$ ) and the CEO's individual near-rational error ( $\varepsilon$ ).

Figure 3 plots the weights on prior and price information against  $\sigma_\varepsilon$ , the standard deviation of the near-rational error. First, note that the weights chosen in equilibrium converge to the constrained efficient weights as  $\sigma_\varepsilon$  goes to zero. Second, the plots reconfirm the result in Proposition 3 that  $\beta_0$  and  $\beta_1$  chosen in equilibrium are inefficiently high for any value of  $\sigma_\varepsilon$ .

The fact that the CEO chooses the weights in the capital investment decision inefficiently affects the ex-ante distribution of capital. The effects on  $E_0[k]$ ,  $V_0[k]$ , and  $\text{Cov}(\theta_a + \theta_f, k)$  are depicted in Figure 4. Note that from the definition of  $k$ , it follows

<sup>21</sup>Note that the social planner's objective is proportional to  $E_0[Y - C]$ , whereas the CEO's objective is proportional to  $\mathcal{E}_{\text{CEO}}[Y - C]$ .

that these three moments (in the benchmark equilibrium) are given by

$$E_0[k] = \beta_0 \quad (33)$$

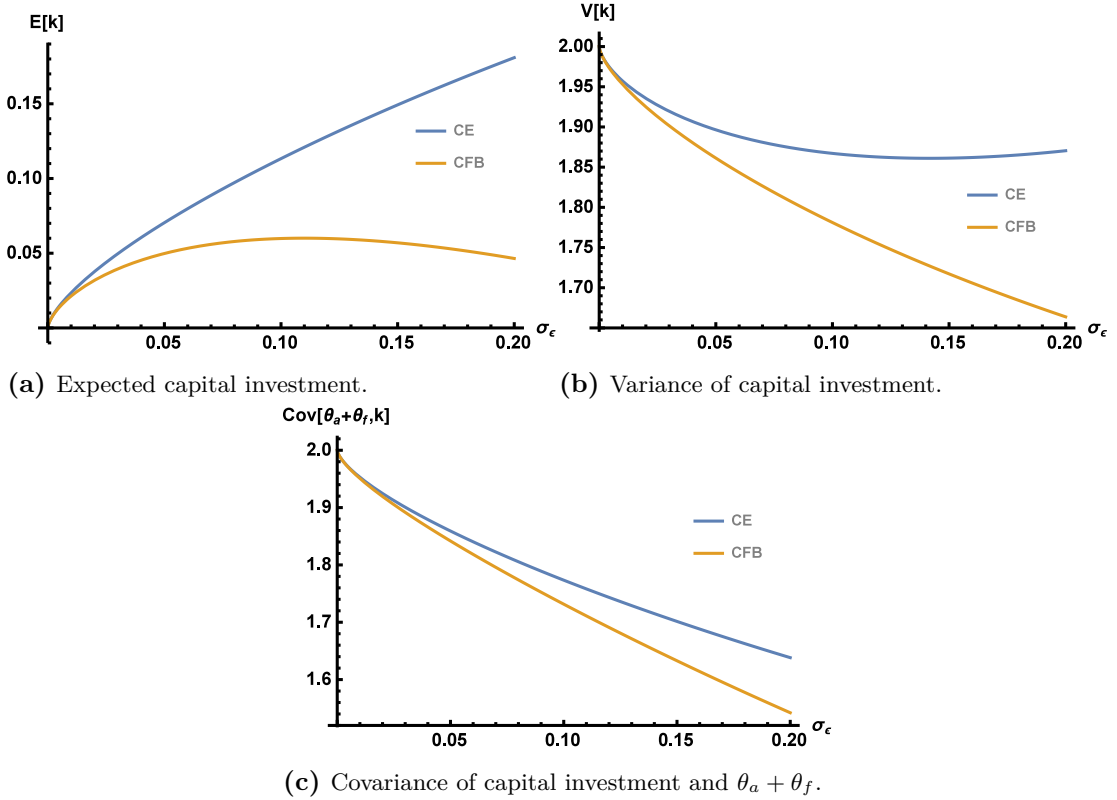
$$V_0[k] = \frac{\alpha_1^2 \beta_1^2}{(1 - \alpha_2)^2} \sigma_a^2 + \left(1 + \frac{\beta_1}{(1 - \alpha_2)}\right)^2 \sigma_\varepsilon^2 + \sigma_f^2 \quad (34)$$

$$\text{Cov}(\theta_a + \theta_f, k) = \frac{\beta_1}{(1 - \alpha_2)} \sigma_\varepsilon^2 + \sigma_f^2. \quad (35)$$

From (33), it then immediately follows that the CEO invests too much on average (compared to the constrained efficient benchmark) because  $\beta_0$  is too high in the competitive equilibrium (see Proposition 2). Moreover, the fact that  $\beta_1$  is inefficiently large increases both the ex-ante variance of (log) capital investment and its covariance with the composite productivity shock.

**Corollary 2** *Relative to the constrained efficient investment rule,  $E_0[k]$ ,  $V_0[k]$ , and  $\text{Cov}(\theta_a + \theta_f, k)$  are inefficiently large.*

**Proof:** See Appendix A.1.10.



**Figure 4:** Expected log capital investment, its ex-ante variance, and the covariance between  $k$  and both productivity shocks in the equilibrium vs. the constrained first-best. Parameters:  $\sigma_a = \sigma_f = \sigma_x = 1$ .

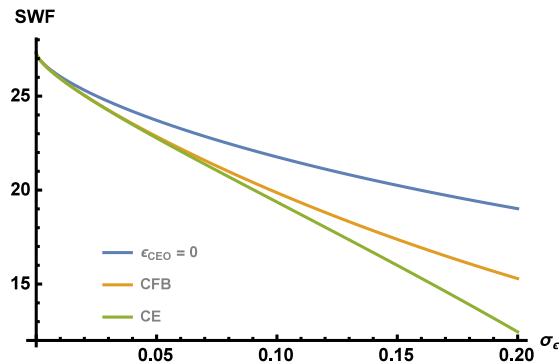
From (34), one can see that the CEO's near-rationality affects  $V_0[k]$  in two ways. (i) It immediately renders  $k$  more volatile because the CEO's expectation is directly exposed to  $\varepsilon$ . Because  $\sigma_\varepsilon$  is assumed to be small (in the sense that agents are *near-rational*), this channel is relatively unimportant in equilibrium. (ii) The CEO's near-rational error is correlated with the equilibrium stock price and therefore with  $\hat{q}$ . This correlation is reflected in the term  $\frac{\beta_1}{(1-\alpha_2)}$  in 34. Thus, the CEO's near-rational error is amplified and renders capital more volatile. This amplification is particularly strong if the CEO and all households rely heavily on price information, that is, if  $\beta_1$  and  $\alpha_2$  are close to one.

Figure 4 plots  $E_0[k]$ ,  $V_0[k]$ , and  $\text{Cov}(k, \theta_a + \theta_f)$  against  $\sigma_\varepsilon$  for one set of parameters. Two interesting observations can be made. (i) The differences between the competitive equilibrium (blue line) and the constrained first-best (orange line) are sizeable for larger values of  $\sigma_\varepsilon$ . (ii) The distortion vanishes as  $\sigma_\varepsilon$  converges to zero, that is, as all agents become fully rational.

Note that the CEO's inefficient use of information is solely related to his own near-rational behavior. In other words, if the CEO became perfectly rational such that he made his investment decision under the fully rational expectation operator, the equilibrium outcome would be constrained efficient.

**Proposition 4** *If the CEO becomes fully rational, the equilibrium outcome is constrained efficient.*

*Proof:* See Appendix A.1.11.



**Figure 5:** Household welfare in a setting with a fully-rational CEO, the constrained first-best, and the benchmark equilibrium. Parameters:  $\sigma_a = \sigma_f = \sigma_x = 1$ .

Figure 5 plots household welfare against the standard deviation of the near-rational error. The blue line corresponds to the setting where the CEO behaves fully rational. The

orange line describes the constrained efficient allocation, and the green line represents the benchmark equilibrium. The vertical distance between the green and the blue line equals the total social cost associated with the CEO's near-rational behavior as plotted in Figure 2. This cost consists of two components: (i) the presence of  $\varepsilon$  in the CEO's investment decision (difference between blue and orange) and (ii) the CEO's inefficient use of information (difference between orange and green).

### 3.4 Constrained First-Best: Limited Discretion

In the following, I explain how a different compensation mechanism can improve welfare. Therefore, I extend the CEO's compensation contract in the benchmark economy with an additional component. This additional component ensures the CEO's evaluation does not deviate too far from the market's evaluation (i.e. the firm's stock price) of the firm's prospects. As a result, the CEO's terminal compensation is a function of both the terminal dividend  $D$  and the current stock price  $P$  (or, equivalently,  $\hat{q}$ ). For parsimony, I use the following functional form:<sup>22</sup>

$$W_{\text{CEO}} = D + (\exp(\delta_0 + \delta_1 \hat{q} + \gamma(\theta_a + \theta_f)) - \exp(\theta_a + \theta_f)) K. \quad (36)$$

As before, the CEO chooses capital investment  $K$  to maximize his expected wealth  $W_{\text{CEO}}$ . Then, optimal log capital investment is given by

$$k = \delta_0 + \delta_1 \hat{q} + \gamma(\beta_0 + \beta_1 \hat{q} + \varepsilon + \theta_f), \quad (37)$$

where  $\{\beta_0, \beta_1\}$  are given in (20) and (21), and  $\{\delta_0, \delta_1, \gamma\}$  are constants (from the CEO's perspective) chosen by the households at  $t = 0$ .<sup>23</sup>

Intuitively, the expression in (37) shows the CEO's capital investment is a combination of two components: (i)  $\beta_0 + \beta_1 \hat{q} + \varepsilon + \theta_f$ , that is, his optimal investment under full discretion, and (ii)  $\delta_0 + \delta_1 \hat{q}$ , that is, a fixed investment rule that depends only on the stock price. The constant  $\gamma$  determines the relative weight of these two components.

The functional form for  $k$  nests two special cases: (i) the benchmark equilibrium with parameters  $\delta_0 = \delta_1 = 0$  and  $\gamma = 1$ , and (ii) the constrained first-best under full

<sup>22</sup>In this section, I assume the CEO's compensation payment is negligibly small compared to the terminal firm value such that the firm's terminal dividend is equal to  $D = Y - C$ .

<sup>23</sup>Note that all households are identical at  $t = 0$ . As a result, no heterogeneity exists across households regarding the optimal values for  $\{\delta_0, \delta_1, \gamma\}$ .



discretion with parameters  $\delta_0 = b_0 - \beta_0$ ,  $\delta_1 = b_1 - \beta_1$ , and  $\gamma = 1$ . Thus, it is obvious that limited discretion weakly dominates both the benchmark equilibrium and the constrained first-best under full discretion in terms of welfare.

At  $t = 0$ , the firm's initial owners (the households) can determine the optimal combination of managerial discretion and the fixed investment rule. Therefore, the weights  $\{\gamma, \delta_0, \delta_1\}$  are chosen ex ante to maximize social welfare  $E_0[W_i]$  given all agents' private information, near-rationality, and equilibrium policies at  $t = 1$ . The optimal compensation structure is presented in the following proposition.

**Proposition 5** *The optimal values for  $\{\gamma, \delta_0, \delta_1\}$  are*

$$\gamma = \frac{\alpha_1^2 \sigma_a^2 \sigma_f^2 + \sigma_\varepsilon^2 (\sigma_f^2 - \alpha_1 \sigma_a^2)}{\alpha_1^2 \sigma_a^2 (\sigma_f^2 + \sigma_\varepsilon^2) + \sigma_f^2 \sigma_\varepsilon^2} \quad (38)$$

$$\delta_0 = \frac{\sigma_a^2 + \sigma_f^2 - V_0[k] - \gamma \beta_0}{2} \quad (39)$$

$$\delta_1 = \frac{\sigma_\varepsilon^6 (\alpha_1 \sigma_a^2 - \sigma_f^2) + \alpha_1^2 \sigma_a^2 \sigma_\varepsilon^4 ((\alpha_1 + 1) \sigma_a^2 - \sigma_f^2)}{(\alpha_1^2 \sigma_a^2 + \sigma_\varepsilon^2) (\alpha_1^2 \sigma_x^2 + \sigma_\varepsilon^2) (\alpha_1^2 \sigma_a^2 (\sigma_f^2 + \sigma_\varepsilon^2) + \sigma_f^2 \sigma_\varepsilon^2)}, \quad (40)$$

where  $V_0[k]$  denotes the ex-ante variance of log capital investment given explicitly in the Appendix.

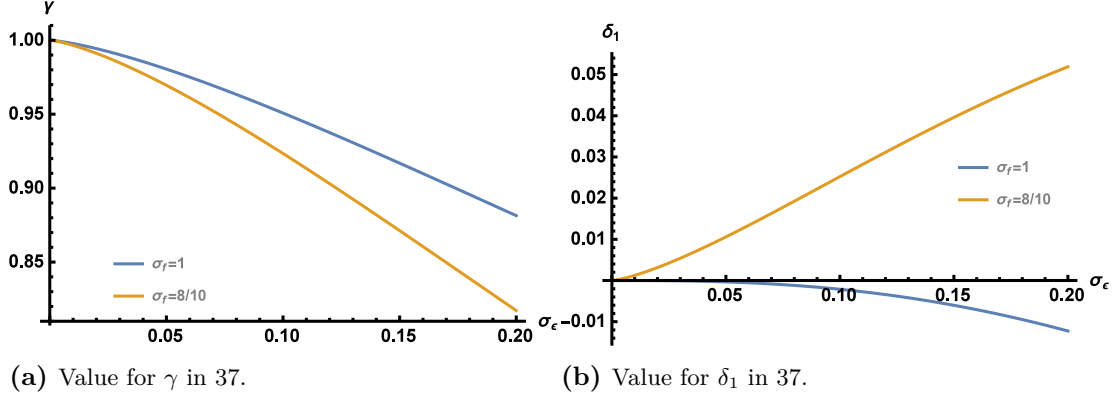
**Proof:** See Appendix A.1.12.

The above system of equations implicitly defines the optimal weights  $\{\gamma, \delta_0, \delta_1\}$  given the agents' equilibrium expectations, namely,  $\{\alpha_1, \alpha_2, \beta_0, \beta_1\}$ . Equation (38) shows that the share  $\gamma$  dedicated to discretionary investment is equal to 1 if and only if  $\sigma_f \rightarrow \infty$ . In this particular case, the CEO's private information about  $\theta_f$  is extremely valuable such that (combined) aggregate capital investment should fully depend on the CEO's choice of  $K$ . As  $\sigma_f$  decreases, the importance of  $\theta_f$  declines and more weight is allocated to the fixed rule component; that is,  $\gamma$  decreases. This result is formally stated in the following corollary.

**Corollary 3** *The weight attached to discretionary investment increases in the standard deviation of  $\theta_f$ :*

$$\frac{\partial \gamma}{\partial \sigma_f} > 0. \quad (41)$$

*Proof:* See Appendix A.1.13.

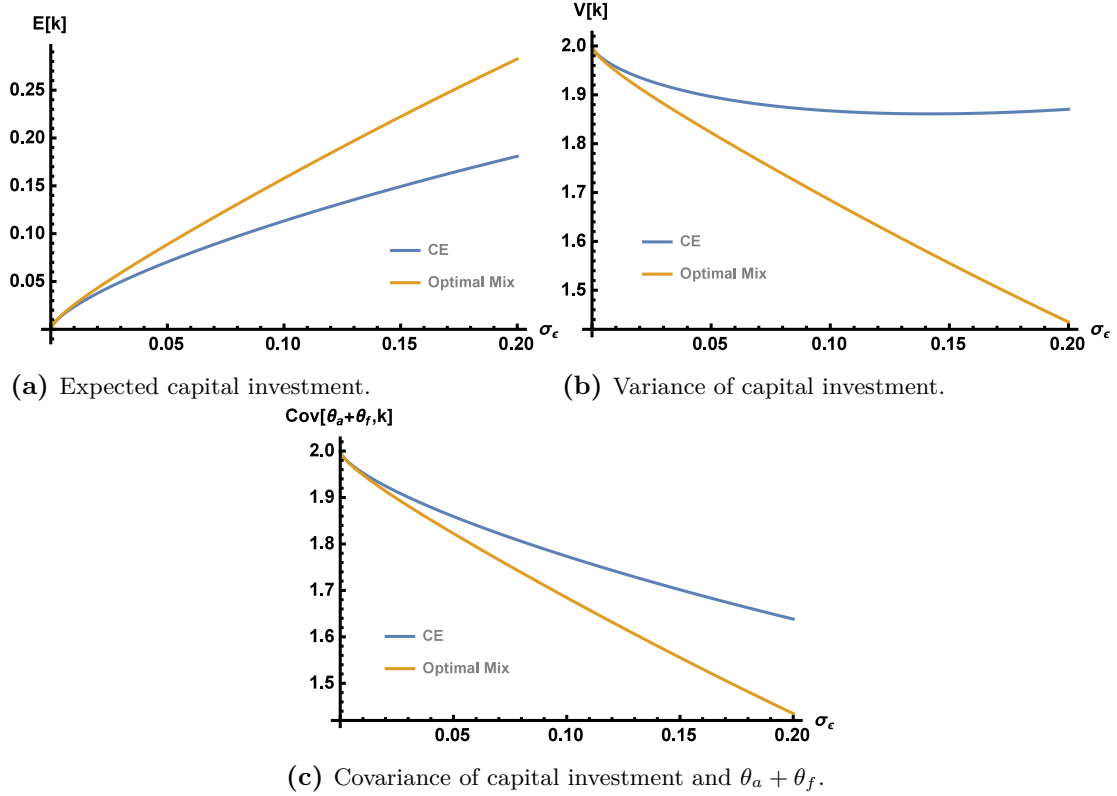


**Figure 6:** Parameters:  $\sigma_a = \sigma_x = 1$  and  $\sigma_f \in \{1, 8/10\}$ .

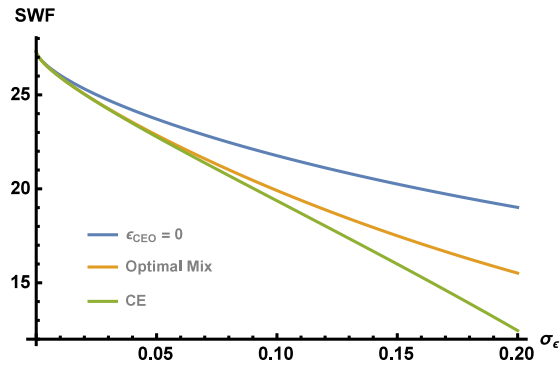
The numerical values for  $\gamma$  and  $\delta_1$  are depicted in Figure 6 for different parameter values. The first panel plots  $\gamma$  for two different values of  $\sigma_f$ . It becomes apparent that (i) more weight is attached to discretionary investment if  $\theta_f$  is more volatile (blue line) and (ii)  $\gamma$  is decreasing in  $\sigma_\epsilon$ . The second effect can be explained as follows: as the agents' near-rational error becomes more volatile, the efficiency loss resulting from discretionary investment increases such that capital investment should be less exposed to the CEO's decision. From the second panel in Figure 6, one can see that the weight on price information in the fixed-rule component is also decreasing in  $\sigma_\epsilon$ . Moreover, this weight can even be negative if  $\sigma_f$  is sufficiently volatile.

Figure 7 plots  $E_0[k]$ ,  $V_0[k]$ , and  $\text{Cov}(\theta_a + \theta_f, k)$  against  $\sigma_\epsilon$ . It can be seen that under the optimally designed combination of discretion and the fixed rule, expected capital investment is higher than in the benchmark equilibrium. At the same time, capital investment is less volatile and also has a lower covariance with the composite productivity shock.

Figure 8 plots household welfare for the optimal investment rule in (37) and (38)-(40) for different parameter values (orange line). Again, the blue line corresponds to the case in which the CEO invests fully rationally, whereas the green line represents welfare in the benchmark equilibrium. For all three cases, welfare is decreasing in  $\sigma_\epsilon$  and it converges to the (unconstrained) first-best as  $\sigma_\epsilon$  goes to zero. Furthermore, it can be observed that the optimal combination of discretion and the fixed rule can reduce some of the social cost that the CEO's near-rational behavior causes.



**Figure 7:** Expected log capital investment, its ex-ante variance, and the covariance between  $k$  and both productivity shocks in the equilibrium vs. the optimal combination of discretion and the fixed rule. Parameters:  $\sigma_a = \sigma_f = \sigma_x = 1$ .



**Figure 8:** Household welfare in a setting with a fully rational CEO, the optimal combination of discretion and the fixed rule, and the benchmark equilibrium. Parameters:  $\sigma_a = \sigma_f = \sigma_x = 1$ .

## 4 Quantitative Model

In this section, the effects studied in the static model are quantified. For that purpose, I use a de-centralization of the model by Croce (2014) as in Hassan and Mertens (2014b).

## 4.1 Setup

Technology is characterized by

$$Y_t = K_t^\alpha (e^{a_t} N_t)^{1-\alpha}, \quad (42)$$

where  $Y_t$  represents output of the consumption good. The productivity shock  $a_t$  has a long-run component  $\omega$  and a short-run component  $\varphi$ ,<sup>24</sup>

$$\Delta a_{t+1} = \mu_a + \omega_t + \varphi_{t+1}, \quad (43)$$

where the long-run component follows an autoregressive process:

$$\omega_t = \rho\omega_{t-1} + \theta_t. \quad (44)$$

As in the static model, I decompose the shock to long-run productivity into two independent components:  $\theta_t = \theta_{a,t} + \theta_{f,t}$ . Both of these shocks are normally distributed with zero mean and variances  $\chi\sigma_\theta^2$  and  $(1-\chi)\sigma_\theta^2$ , respectively, such that the parameter  $\chi \in [0, 1]$  controls the importance of inside ( $\theta_f$ ) and outside information ( $\theta_a$ ).<sup>25</sup>

The firm accumulates capital according to the following law of motion:

$$K_{t+1} = (1 - \delta_k + G_t)K_t, \quad (45)$$

where  $\delta_k$  is the rate of depreciation. Convex adjustment costs to capital occur following Jermann (1998):

$$G_t = \nu_0 + \frac{\nu_1}{1 - \frac{1}{\xi}} \left( \frac{I_t}{K_t} \right)^{1 - \frac{1}{\xi}}, \quad (46)$$

where  $\{\nu_0, \nu_1\}$  are positive constants and  $\xi$  determines the equilibrium elasticity of the capital stock with respect to the CEO's private marginal  $q$ .

A representative firm purchases labor services from households. As a result, labor is paid its marginal product such that the equilibrium wage is equal to  $w_t = (1 - \alpha)\frac{Y_t}{N_t}$ . Households own the stock of capital and rent it to the firm in return for the periodic dividend (per unit of capital)  $D_t = \alpha\frac{Y_t}{K_t} - \frac{I_t}{K_t}$ . As a result, the return per unit of capital

<sup>24</sup>For simplicity, I ignore the short-run component in the following; that is, I set the variance of  $\varphi$  to zero.

<sup>25</sup>Note that, for example,  $\chi = 0$  implies that only inside information ( $\theta_f$ ) determines variation in  $\theta_t$ , whereas  $\chi = 1$  implies that only outside information ( $\theta_a$ ) determines variation in  $\theta_t$ .

is given by

$$R_{t+1} = \frac{(1 - \delta_k + G_{t+1})Q_{t+1} + D_{t+1}}{Q_t}. \quad (47)$$

The CEO is endowed with a fixed fraction of the firm's shares initially. He therefore receives a periodic payment proportional to the firm's dividend. For simplicity, I assume this periodic compensation is negligibly small such that it does not affect economic aggregates. As a result, he maximizes the expected stream of future dividends using his stochastic discount factor  $M_{\text{CEO},t+1}$ . I solve for the firm's optimal investment in Appendix B.1 but note that the CEO's optimality condition for investment requires that

$$\mathcal{E}_{\text{CEO},t} \left[ M_{\text{CEO},t+1} R_{t+1}^I \right] = 1, \quad (48)$$

where  $\mathcal{E}_{\text{CEO},t}$  denotes the CEO's near-rational expectation operator and  $R_{t+1}^I$  denotes the return on investment.

I assume the CEO's investment decision at date  $t$  depends on date  $t - 1$  information. This assumption is important because it allows the CEO to have superior information about future productivity vis-a-vis the households. Without this assumption, the households could infer the CEO's inside information from his (publicly observable) capital investment decision. Intuitively, I assume that the CEO's investment decision is made one instant before the private signals are realized and the stock price is determined.

An individual household  $i \in [0, 1]$  has Epstein and Zin (1989) preferences over the consumption bundle  $\tilde{C}_{it}$ :

$$U_{it} = \left( (1 - \delta)\tilde{C}_{it}^{1-\frac{1}{\xi}} - \pi(b_{it}) + \delta \mathcal{E}_{it} \left[ U_{it+1}^{1-\gamma} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}}, \quad (49)$$

where the parameters  $\psi$  and  $\gamma$  measure the household's intertemporal elasticity of substitution and relative risk aversion, respectively.  $\pi(b_{it})$  is a small penalty for holding bonds that ensures a well-defined portfolio choice at the deterministic steady state as in Judd and Guu (2001). The consumption bundle  $\tilde{C}_{it}$  is a CES aggregate of consumption and leisure:

$$\tilde{C}_{it} = \left( oC_{it}^{1-\frac{1}{\xi_i}} + (1 - o)(e^{a_{t+1}}(1 - n_{it}))^{1-\frac{1}{\xi_i}} \right)^{\frac{1}{1-\frac{1}{\xi_i}}}, \quad (50)$$

where leisure is multiplied by aggregate productivity to ensure the existence of a balanced

growth path.

At the beginning of every period, each household receives a private signal about  $\theta_{a,t+1}$  which is a part of the composite shock to long-run productivity:

$$x_{it} = \theta_{a,t+1} + \nu_{it}, \quad (51)$$

where  $\nu_{it}$  is independent across households and normally distributed with zero mean and standard deviation  $\sigma_x$ . As in the static model, I assume the CEO does not receive a private signal about  $\theta_{a,t+1}$  but observes  $\theta_{f,t+1}$  without noise at date  $t$ .

All agents observe the state of the economy at time  $t$  and understand the structure of the economy as well as the equilibrium mapping of dispersed information into prices and economic aggregates. The rational expectations operator conditional on all information available at time  $t$  is

$$E_{it}[\cdot] = E[\cdot | x_{it}, \omega_t, D_t, R_t, P_t, Y_t, N_t, C_t, K_t, I_t, \omega_{t-1}, \mathcal{E}_{\text{CEO},t-1}[\theta_t]]. \quad (52)$$

The only two sources of uncertainty are thus  $\theta_{t+1}$ , the shock to long-run productivity, and  $\mathcal{E}_{\text{CEO},t}[\theta_{t+1}]$ , the manager's date  $t$  expectation of this shock (which affects firm investment at  $t+1$ ). In the following, I show that at date  $t$ , all agents can infer information about  $\theta_{t+1}$  from economic aggregates (e.g. the stock price). Furthermore, the households have to predict the CEO's date  $t$  expectation to forecast his capital investment decision next period.<sup>26</sup>

As in the static model, I assume every agent in the economy makes a small correlated error when forming his expectation about  $\theta_{a,t+1}$ :

$$\mathcal{E}_{it}[\theta_{a,t+1}] = E_{it}[\theta_{a,t+1}] + \varepsilon_t, \quad (53)$$

where, again,  $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ .

Given  $x_{it}$  and their knowledge about the state of the economy, households maximize their lifetime utility by choosing a time path for consumption and labor, and their stock and bond holdings  $\{\tilde{C}_{it}, n_{it}, k_{it}, b_{it}\}_{t=0}^\infty$ . Each household's optimization is subject to a

<sup>26</sup>Without loss of generality, I assume the households observe the CEO's past expectations about  $\theta$ . This additional source of information does not affect the households' equilibrium policies because it does not convey useful information about future productivity.

budget constraint:

$$P_t k_{it+1} + b_{it} = P_{t-1} R_t k_{it} + (1 + r_{t-1}) b_{it-1} + H_{it} - C_{it} + w_t n_{it}, \quad (54)$$

where  $P_t$  is the stock price at date  $t$ ,  $H_{it}$  are transfers from state-contingent claims discussed below, and  $w_t$  is the wage rate.

The market-clearing conditions are given by

$$\int_0^1 k_{it} di = K_t \quad (55)$$

$$\int_0^1 b_{it} di = 0 \quad (56)$$

$$\int_0^1 n_{it} di = N_t \quad (57)$$

$$C_t + I_t = Y_t. \quad (58)$$

The payments from contingent claims  $H_{it}$  allow me to ignore the wealth distribution across households. At the beginning of each period (and before receiving their private signal), households can trade claims contingent on the state of the economy and on the realization of noise in their private signal  $\nu_{it}$ . These claims are in zero net supply and pay off at the beginning of the following period. Because they are traded before any information about  $\theta_{a,t+1}$  is known, their prices cannot reveal any information about future productivity. This trading thus completes markets between periods, without affecting households' signal-extraction problem. In equilibrium, all households choose to hold these securities with net payoff:

$$H_{it} = P_{t-1} R_t - P_{t-1} R_t k_{it} - (1 + r_{t-1}) b_{it-1}, \quad (59)$$

such that all households enter each period with the same amount of wealth. It then follows immediately that these claims are in zero net supply:

$$\int_0^1 H_{it} di = 0. \quad (60)$$

**Definition 4** *Given a time path of shocks  $\{\theta_{a,t}, \theta_{f,t}, \varepsilon_t, \{\nu_{it} \forall i\}\}_{t=0}^{\infty}$ , an equilibrium in this economy is a time path of quantities  $\{\{C_{it}, b_{it}, n_{it}, k_{it} \forall i\}, C_t, N_t, K_t, Y_t, I_t, G_t, a_t, \omega_t\}_{t=0}^{\infty}$ , signals  $\{x_{it} \forall i\}_{t=0}^{\infty}$ , and prices  $\{P_t, r_t, R_t, w_t\}_{t=0}^{\infty}$  with the following properties:*

1.  $\{C_{it}, b_{it}, n_{it}, k_{it}\}_{t=0}^{\infty}$  maximize households' lifetime utility given the vector of prices and the random sequences  $\{\varepsilon_t, \nu_{it}\}_{t=0}^{\infty}$ ;
2. The demand for labor services solves the representative firm's maximization problem given the vector of prices;
3.  $\{I_t\}_{t=0}^{\infty}$  maximizes the CEO's lifetime utility;
4.  $\{w_t\}_{t=0}^{\infty}$  clears the labor market,  $\{P_t\}_{t=0}^{\infty}$  clears the stock market, and  $\{r_t\}_{t=0}^{\infty}$  clears the bond market;
5.  $\{Y_t\}_{t=0}^{\infty}$  is determined by the production function,  $R_t$  from the definition of the stock return, and  $\{\omega_t\}_{t=0}^{\infty}$ ,  $\{a_t\}_{t=0}^{\infty}$ ,  $\{K_t\}_{t=0}^{\infty}$ , and  $\{G_t\}_{t=0}^{\infty}$  evolve according to (43), (44), (45), and (46), respectively.
6.  $\{C_t, N_t\}$  are given by the identities  $C_t = \int_0^1 C_{it} di$  and  $N_t = \int_0^1 n_{it} di$ .

## 4.2 Solution

I use the solution method developed in Mertens (2011) to transform the equilibrium conditions of the model into a form that I can solve with standard techniques. The key to this approach is to show that all prices and economic aggregates are functions of the usual macroeconomic state variables of the model  $S_t = \{K_t, \omega_{t-1}, \theta_t, \mathcal{E}_{\text{CEO}, t-1}[\theta_t]\}$  as well as of the households' average expectation of  $\theta_{t+1}$ :

$$\hat{q}_t = \int_0^1 \mathcal{E}_{it}[\theta_{t+1}] di. \quad (61)$$

**Lemma 6** *A recursive equilibrium exists satisfying the system of equations in Definition 4 with the following properties:*

1. A household's optimal behavior depends on the current (commonly known) state of the economy,  $S_t$ , the households' conditional expectation of the next period's innovation to productivity,  $\mathcal{E}_{it}[\theta_{t+1}]$ , and the average expectation of this innovation  $\hat{q}_t$ . The conditional expectation, in turn, depends on the private signal  $x_{it}$  as well as  $\hat{q}_t$ . I can thus write the set of state variables that determine individual behavior



as

$$z_{it} = z(S_t, \hat{q}_t, \mathcal{E}_{it}[\theta_{t+1}]), z = c, n, k, b. \quad (62)$$

2. All prices and economic aggregates depend on the current state of the economy and  $\hat{q}_t$ :

$$Z_t = Z(S_t, \hat{q}_t), Z = C, N, P, r, R. \quad (63)$$

**Proof:** See Appendix A.1.14.

Given this lemma, I am able to use standard perturbation methods to solve for the households' equilibrium policies as a function of  $\{S_t, \hat{q}_t, \mathcal{E}_{it}[\theta_{t+1}]\}$  and for all economic aggregates as a function of  $\{S_t, \hat{q}_t\}$ . Thus, I can separate the solution of the non-linear dynamic model from the information microstructure by simply taking  $\mathcal{E}_{it}[\theta_{t+1}]$  and  $\hat{q}_t$  as state variables. The final step of the solution is then to solve for  $\mathcal{E}_{it}[\theta_{t+1}]$  and  $\hat{q}_t = \int_0^1 \mathcal{E}_{it}[\theta_{t+1}] di$ .

**Condition 2** *The equilibrium stock price  $P_t$  or at least one other economic aggregate or price is a strictly monotonic function of  $\hat{q}_t$ .*

A direct consequence of Lemma 6 is that all prices and economic aggregates have the same information content. Given Condition 2,  $\hat{q}_t$  is simply a monotonic transformation of  $P_t$  that allows the households and the CEO to infer  $\hat{q}_t$  from observing the equilibrium stock price. Learning from the stock price is then just as good as learning from its monotonic transformation  $\hat{q}_t$ . Although I cannot solve for the mapping of  $\hat{q}_t$  into  $P_t$  in closed form, I can easily check for monotonicity using the numerical solution of the model.

**Lemma 7** *The agents' (rational) equilibrium expectations of  $\theta_{t+1}$  are independent of the aggregate dynamics of the model and can be written as  $E_{it}[\theta_{t+1}] = \alpha_1 x_{it} + \alpha_2 \hat{q}_t$  with*

$$\alpha_1 = \frac{\sigma_x^{-2}}{\sigma_a^{-2} + \sigma_x^{-2} + \alpha_1^2 \sigma_\varepsilon^{-2}} \quad (64)$$

$$\alpha_2 = \frac{\alpha_1^2 \sigma_\varepsilon^{-2}}{\sigma_x^{-2} + \alpha_1^2 \sigma_\varepsilon^{-2}}, \quad (65)$$

and for the CEO as  $E_{CEO,t}[\theta_{t+1}] = \beta_1 \hat{q}_t + \theta_{f,t+1}$  with

$$\beta_1 = \frac{(1 - \alpha_2)}{\alpha_1} \frac{\alpha_1^2 \sigma_\varepsilon^{-2}}{(\sigma_a^{-2} + \alpha_1^2 \sigma_\varepsilon^{-2})}. \quad (66)$$

**Proof:** See Appendix A.1.15.

Therefore, the agents' equilibrium expectations take the same form as in the static model.

### 4.3 Results

Parameter	Value	Parameter	Value
$\alpha$	0.34	$\delta_k$	0.06
$\delta$	0.96	$\gamma$	10
$\mu$	0.018	$\psi$	2
$o$	0.20	$\xi$	4
$\rho$	0.95	$\sigma_\theta$	0.015
$\sigma_\varepsilon/\sigma_\theta$	0.001	$\sigma_x/\sigma_\theta$	1
$\xi_l$	1	$\gamma_{CEO}$	3

**Table 1:** Calibrated Parameters. Note:  $\alpha$ : capital share;  $\delta_k$ : capital depreciation rate;  $\delta$ : subjective discount factor;  $\gamma$ : relative risk aversion;  $\mu$ : average productivity growth;  $\psi$ : intertemporal elasticity of substitution;  $o$ : consumption share;  $\xi$ : adjustment cost parameter;  $\rho$ : auto-regressive coefficient long-run component;  $\sigma_\theta$ : standard deviation of  $\theta_t$ ;  $\sigma_\varepsilon$ : standard deviation of near-rational error;  $\sigma_x$ : standard deviation of noise in private signal;  $\xi_l$ : consumption bundle elasticity;  $\gamma_{CEO}$ : CEO's risk-aversion coefficient.

Table 1 lists the parameters used to evaluate the quantitative model. I set all the standard *macroeconomic* parameters equal to the values used in Croce (2014). For instance,  $\alpha$  is calibrated to match the capital income share and  $\delta_k$  to match the annualized depreciation rate in the US economy. The average annual productivity growth rate  $\mu$  is 1.8%. I set the relative risk-aversion coefficient  $\gamma$  to a moderate level of 10 and the IES to a value of 2. The annualized subjective discount factor  $\delta$  is set to a value of 0.96. Moreover, the parameters  $\{\nu_0, \nu_1\}$  in the adjustment-cost function are set such that at the deterministic steady state,  $G_t - \frac{I_t}{K_t} = 0$  and  $\partial G_t / \partial (I_t / K_t) = 1$ . Therefore,  $\nu_0 = \left(\frac{1}{1-\xi}\right) (\delta + e^\mu - 1)$  and  $\nu_1 = (\delta + e^\mu - 1)^{\frac{1}{\xi}}$ . I set the standard deviation of the shock to long-run productivity growth to  $\sigma_\theta = 1.5\%$ . The consumption bundle elasticity  $\xi_l$  is set to 1 as in Hassan and Mertens (2014b) and Croce (2014). The CEO's risk-aversion coefficient is set to 3, a value consistent with other studies on CEO risk taking.<sup>27</sup>

I calibrate the values for the three remaining parameters  $\{\sigma_x, \sigma_\varepsilon, \chi\}$  and check for

<sup>27</sup>For example, see Glover and Levine (2014a) and Lewellen (2006). Moreover, I assume this value corresponds to the inverse of the CEO's IES coefficient  $\psi_{CEO}$ .

robustness in section 4.4. The calibrated values for these three parameters are described in turn: (i) I set  $\sigma_x$  equal to  $\sigma_\theta$ . (ii) The standard deviation of the near-rational error is set to 1/1000 times the standard deviation of  $\theta_t$  capturing the underlying idea that the individual cost of the near-rational error is negligibly small. (iii) The parameter  $\chi$  determining the importance of inside versus outside information is set to 0.25.

Table 2 shows the results for the benchmark calibration presented in Table 1. I analyze the welfare losses and macro/financial quantities in three settings: (i) the competitive equilibrium (CE), (ii) the constrained first-best (CFB), and (iii) the economy where the CEO is forced to ignore price information (NPI).<sup>28</sup>

	Data		Model		
	Estimate	Std. Err.	CE	CFB	NPI
$\beta_1$			0.99	0.36	0
$\chi$			0.25	0.25	0.25
Welfare Loss			0.55	–	0.46
$E[r_{ex}]$	4.71	2.17	4.07	4.04	4.10
$\sigma[r_{ex}]$	21.21	1.33	12.44	11.41	12.39
$E[rf]$	0.64	0.34	0.79	1.29	1.07
$\sigma[rf]$	3.82	0.46	2.47	2.09	2.34
$\sigma[dy]$	3.34	0.39	2.47	2.08	2.34
$\sigma[di]$	14.86	0.44	9.48	6.07	8.80
$\sigma[dc]$	2.17	0.04	2.69	2.81	2.80
$\text{Corr}(dc, r_{ex})$	0.25	0.12	0.01	0.16	-0.07
$\text{ACF}[dc]$	0.51	0.14	0.69	0.55	0.59
$\text{ACF}[r_{ex}]$	0.05	0.11	0.25	0.25	0.22
$\text{ACF}[rf]$	0.7	0.08	0.55	0.13	0.19

**Table 2:** Notation: CE: competitive equilibrium; CFB: constrained first-best; NPI: no price information;  $\beta_1$ : weight on price information in three cases;  $\chi$ : relative importance of inside vs outside information;  $r_{ex}$ : log excess stock return;  $rf$ : log risk-free rate;  $dy, di, dc$ : first difference in log output, investment, and consumption; ACF: first-order auto-correlation. Data values based on annual US data 1929-2008. All values in the second panel are in %.

Just as in the simple model, the CEO excessively uses price information in equilibrium. This distortion can be inferred from the first row where I present the weight  $\beta_1$  attached to price information. This weight is approximately equal to 99% in equilibrium, whereas the social planner reduces this weight to 36% in the constrained first-best to maximize household welfare. The CEO's excessive use of information translates into a loss of 0.55%

<sup>28</sup>See Appendix B.2 for details on the construction of the constrained first-best. The computation of the welfare loss is explained in Appendix B.3.

of permanent consumption relative to the constrained first-best. The model performs well on matching the empirical moments. Regarding the excess return, one can see that both its expectation and standard deviation are lower in the constrained first-best. Hence, the CEO's inefficient use of price information leads to higher and more volatile stock returns. Similarly, output growth is more volatile in the competitive equilibrium. The effect is particularly strong for investment growth, which is roughly 1.5 times more volatile in the competitive equilibrium.

The last column shows the corresponding results in an economy where the CEO is forced to ignore price information. In that case, the welfare loss relative to the constrained first-best is given by 0.46%. Interestingly, this loss is smaller than that in the competitive equilibrium. This finding implies that if households can choose between this scenario and the competitive equilibrium, they would rather have the CEO ignoring information revealed by the stock price.

#### 4.4 Robustness

In this section, I conduct comparative statics with respect to  $\chi$  and  $\sigma_x$ . Table 3 shows the model-implied moments for  $\chi \in \{0.15, 0.35\}$ . One can see that the three important results from the benchmark calibration survive: (i) too much weight on price information, (ii) sizeable welfare loss, and (iii) excessively volatile returns and investment growth.

The same pattern can be observed in Table 4, which displays the results for  $\sigma_x/\sigma_\theta \in \{1/5, 5\}$ .

## 5 Conclusion

This paper analyzes the efficiency of stock-based compensation in the presence of noisy but informative stock prices.

In my model, each household receives a private signal about future productivity. If all agents behave fully rationally, the equilibrium stock price aggregates this information without noise such that capital is allocated perfectly. In this case, rewarding the CEO with stock-based compensation is efficient because it aligns his utility with that of the average household. However, if I allow for the possibility that all agents make a small correlated error when forming their expectations, stock-based compensation does not

	Model ( $\chi = 0.15$ )			Model ( $\chi = 0.35$ )		
	CE	CFB	NPI	CE	CFB	NPI
$\beta_1$	0.99	0.36	0	0.99	0.36	0
$\chi$	0.15	0.15	0.15	0.35	0.35	0.35
Welfare Loss	0.55	–	0.50	0.55	–	0.42
$E[r_{ex}]$	2.67	2.64	2.69	5.47	5.45	5.52
$\sigma[r_{ex}]$	12.58	11.48	12.55	12.26	11.33	12.20
$E[rf]$	2.76	2.79	2.75	1.25	1.28	1.24
$\sigma[rf]$	0.80	1.29	0.98	0.79	1.29	1.15
$\sigma[dy]$	2.48	2.12	2.41	2.47	2.06	2.28
$\sigma[di]$	9.41	6.06	9.00	9.55	6.06	8.59
$\sigma[dc]$	2.74	2.85	2.80	2.64	2.78	2.81
$\text{Corr}(dc, r_{ex})$	-0.02	0.17	-0.06	0.03	0.13	-0.09
$\text{ACF}[dc]$	0.71	0.57	0.65	0.69	0.54	0.54
$\text{ACF}[r_{ex}]$	0.26	0.27	0.25	0.25	0.24	0.21
$\text{ACF}[rf]$	0.59	0.16	0.33	0.53	0.10	0.09

**Table 3:** Notation: CE: competitive equilibrium; CFB: constrained first-best; NPI: no price information;  $\beta_1$ : weight on price information in three cases;  $\chi$ : relative importance of inside vs. outside information;  $r_{ex}$ : log excess stock return;  $rf$ : log risk-free rate;  $dy, di, dc$ : first difference in log output, investment, and consumption; ACF: first-order auto-correlation. All values in the second panel are in %. All model parameters except  $\chi$  are taken from Table 1.

prevent the CEO from committing this error: his private cost of investing near-rationally is orders of magnitude smaller than the implied social cost.

I show that the CEO’s exposure to this near-rational error leads to an inefficient use of information. Conditional on the CEO behaving near-rationally, the social planner would like to limit his weight on price information.

In the dynamic model, I implement this mechanism into a production-based asset pricing model. I show that the main conclusions from the static model remain valid. In particular, the CEO overuses price information in the competitive equilibrium, which leads to a sizeable welfare loss of over 0.5% of permanent consumption. Moreover, both excess returns and investment growth are excessively volatile. Interestingly, in my preferred calibration, households are better off if the CEO fully ignores price information in equilibrium.

	Model ( $\sigma_x/\sigma_\theta = 1/5$ )			Model ( $\sigma_x/\sigma_\theta = 5$ )		
	CE	CFB	NPI	CE	CFB	NPI
$\beta_1$	0.99	0.36	0	0.99	0.36	0
$\chi$	0.25	0.25	0.25	0.25	0.25	0.25
Welfare Loss	0.55	–	0.46	0.51	–	0.45
$E[r_{ex}]$	4.07	4.05	4.11	4.11	4.07	4.11
$\sigma[r_{ex}]$	12.52	11.50	12.47	12.17	11.14	12.11
$E[rf]$	2.41	2.44	2.39	1.15	1.17	1.13
$\sigma[rf]$	0.79	1.29	1.07	0.85	1.31	1.07
$\sigma[dy]$	2.47	2.09	2.34	2.45	2.09	2.35
$\sigma[di]$	9.51	6.08	8.80	9.19	6.12	8.80
$\sigma[dc]$	2.69	2.81	2.80	2.71	2.83	2.81
$\text{Corr}(dc, r_{ex})$	0.01	0.15	-0.08	0.00	0.17	-0.04
$\text{ACF}[dc]$	0.70	0.55	0.59	0.68	0.55	0.60
$\text{ACF}[r_{ex}]$	0.25	0.24	0.21	0.29	0.31	0.28
$\text{ACF}[rf]$	0.57	0.18	0.18	0.44	0.13	0.21

**Table 4:** Notation: CE: competitive equilibrium; CFB: constrained first-best; NPI: no price information;  $\beta_1$ : weight on price information in three cases;  $\chi$ : relative importance of inside vs. outside information;  $r_{ex}$ : log excess stock return;  $rf$ : log risk-free rate;  $dy, di, dc$ : first difference in log output, investment, and consumption; ACF: first-order auto-correlation. All values in the second panel are in %. All model parameters except  $\sigma_x/\sigma_\theta$  are taken from Table 1.

## References

- AKERLOF, G., AND J. YELLEN (1985): “A Near-Rational Model of the Business Cycle with Wage and Price Inertia,” *Quarterly Journal of Economics*.
- ANGELETOS, G.-M., G. LORENZONI, AND A. PAVAN (2010): “Beauty Contests and Irrational Exuberance: A Neoclassical Approach,” *Working Paper*.
- ANGELETOS, G.-M., AND A. PAVAN (2007): “Efficient Use of Information and Social Value of Information,” *Econometrica*, 75(4), 1103–1142.
- ANGELETOS, G.-M., AND A. PAVAN (2009): “Policy with Dispersed Information,” *Journal of the European Economic Association*, 7(1), 11–60.
- BAKKE, T.-E., AND T. M. WHITED (2010): “Which Firms Follow the Market? An Analysis of Corporate Investment Decisions,” *Review of Financial Studies*, 23(5), 1941–1980.
- BENMELECH, E., E. KANDEL, AND P. VERONESI (2010): “Stock-based Compensation and CEO (Dis)Incentives,” *The Quarterly Journal of Economics*, 125(4), 1769–1820.
- BOND, P., AND I. GOLDSTEIN (2014): “Government Intervention and Information Aggregation by Prices,” *Journal of Finance*, forthcoming.
- BRUNNERMEIER, M., A. SIMSEK, AND W. XIONG (2014): “A Welfare Criterion for Models with Distorted Beliefs,” *Quarterly Journal of Economics*, forthcoming.
- CHEN, Q., I. GOLDSTEIN, AND W. JIANG (2007): “Price Informativeness and Investment Sensitivity to Stock Prices,” *Review of Financial Studies*, 20(3), 619–650.
- CHETTY, R. (2012): “Bonds on Elasticities with Optimization Frictions: A Synthesis of Micro and Macro Evidence on Labor Supply,” *Econometrica*, 80(3), 969–1018.
- COCHRANE, J. H. (1989): “The Sensitivity of Tests of the Intertemporal Allocation of Consumption to Near-Rational Alternatives,” *The American Economic Review*, 79(3), 319–337.
- CROCE, M. M. (2014): “Long-run Productivity Risk: A New Hope for Production-based Asset Pricing?,” *Journal of Monetary Economics*, 66, 13–31.

- DIAMOND, D. W., AND R. E. VERRECCHIA (1981): “Information Aggregation in a Noisy Rational Expectations Economy,” *Journal of Financial Economics*, 9(3), 221–235.
- DIAMOND, D. W., AND R. E. VERRECCHIA (1982): “Optimal Managerial Contracts and Equilibrium Security Prices,” *The Journal of Finance*, 37(2), 275–287.
- EDMANS, A., I. GOLDSTEIN, AND W. JIANG (2012): “The Real Effects of Financial Markets: The Impact of Prices on Takeovers,” *The Journal of Finance*, 67(3), 933–971.
- EPSTEIN, L. G., AND S. E. ZIN (1989): “Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework,” *Econometrica: Journal of the Econometric Society*, pp. 937–969.
- FOUCALT, T., AND L. FRESARD (2013): “Learning from Peers’ Stock Prices and Corporate Investment,” *Journal of Financial Economics*.
- GLOVER, B., AND O. LEVINE (2014a): “Idiosyncratic Risk and the Manager,” *Working Paper*.
- GLOVER, B., AND O. LEVINE (2014b): “Uncertainty, Investment, and Managerial Incentives,” *Working Paper*.
- GOLDSTEIN, I., E. OZDENOREN, AND K. YUAN (2013): “Trading Frenzies and Their Impact on Real Investment,” *Journal of Financial Economics*, 109(2), 566–582.
- GOLDSTEIN, I., AND L. YANG (2014): “Market Efficiency and Real Efficiency: The Connect and Disconnect via Feedback Effects,” *Working Paper*.
- GROSSMAN, S. (1976): “On the Efficiency of Competitive Stock Markets where Trades have Diverse Information,” *The Journal of Finance*, 31(2), 573–585.
- HASSAN, T. A., AND T. MERTENS (2011): “Market Sentiment: A Tragedy of the Commons,” *The American Economic Review, Papers and Proceedings*, 101(2), 402–405.
- HASSAN, T. A., AND T. M. MERTENS (2014a): “Information Aggregation in DSGE Models,” *NBER Macroeconomics Annual*.
- (2014b): “The Social Cost of Near-Rational Investment,” *Working Paper*.



- HAYEK, F. (1945): “The Use of Knowledge in Society,” *American Economic Review*, 35, 519–530.
- HELLWIG, M. F. (1980): “On the aggregation of information in competitive markets,” *Journal of Economic Theory*, 22(3), 477–498.
- JERMANN, U. J. (1998): “Asset Pricing in Production Economies,” *Journal of Monetary Economics*, 41(2), 257–275.
- JUDD, K. L., AND S.-M. GUU (2001): “Asymptotic Methods for Asset Market Equilibrium Analysis,” *Economic Theory*, 18(1), 127–157.
- LEWELLEN, K. (2006): “Financing Decisions when Managers are Risk Averse,” *Journal of Financial Economics*, 82(3), 551–589.
- LUO, Y. (2005): “Do Insiders Learn from Outsiders? Evidence from Mergers and Acquisitions,” *The Journal of Finance*, 60(4), 1951–1982.
- MERTENS, T. (2011): “Volatile Stock Markets: Equilibrium Computation and Policy Analysis,” *NYU Working Paper*.
- MORELLEC, E. (2004): “Can Managerial Discretion Explain observed Leverage Ratios?,” *Review of Financial Studies*, 17, 257–294.
- MURPHY, K. J. (2012): “Executive Compensation: Where we are, and how we got there,” *Handbook of the Economics of Finance. Elsevier Science North Holland*.
- PENG, L., AND A. ROELL (2014): “Managerial Incentives and Stock Price Manipulation,” *Journal of Finance*, 59(2), 487–526.
- STEIN, J. C. (1989): “Efficient Capital Markets, Inefficient Firms: A Model of Myopic Corporate Behavior,” *Quarterly Journal of Economics*, 104, 655–669.
- STREBULAEV, I. A., AND T. M. WHITED (2011): “Dynamic Models and Structural Estimation in Corporate Finance,” *Foundations and Trends in Finance*, 6, 1–63.
- STROBL, G. (2014): “Stock-based Managerial Compensation, Price Informativeness, and the Incentive to Overinvest,” *Journal of Corporate Finance*.
- SUBRAHMANYAM, A., AND S. TITMAN (2001): “Feedback from Stock Prices to Cash Flows,” *Journal of Finance*, 56, 2389–2413.

SUBRAHMANYAM, A., AND S. TITMAN (2013): “Financial Market Shocks and the Macroeconomy,” *Review of Financial Studies*, 26(11), 2687–2717.

TILLE, C., AND E. VAN WINCOOP (2014a): “International Capital Flows under Dispersed Private Information,” *Journal of International Economics*, 93(1), 31–49.

——— (2014b): “Solving DSGE Portfolio Choice Models with Dispersed Private Information,” *Journal of Economic Dynamics and Control*, 40, 1–24.

# A Technical Appendix

## A.1 Proofs

### A.1.1 Proof of Lemma 1

First note that the market-clearing condition can be written as

$$Pr(\mathcal{E}_i[D] \leq P) = \frac{\bar{\omega} - 1}{\bar{\omega} - \underline{\omega}}. \quad (\text{A.1})$$

Next, the expected dividend can be written as  $\mathcal{E}_i[D] = (1 - \omega_{\text{CEO}})(\mathcal{E}_i[Y] - \frac{1}{2}\mathcal{E}_i[K^2])$ . Because the households do not use their private signal to forecast  $K$ , no heterogeneity in  $\mathcal{E}_i[K]$  exists, such that I write  $\mathcal{E}[K]$  in the following. Moreover, the conditional expectation of  $Y$  can be written as

$$\begin{aligned} \mathcal{E}_i[Y] &= \exp\left(\mathcal{E}_i[y] + \frac{1}{2}V_1[y]\right) = \exp\left(\mathcal{E}_i[\theta_a] + \beta_0 + \beta_1\hat{q} + \frac{1}{2}V_1[y]\right) \\ &= \exp\left(\hat{q} + \alpha_1\nu_i + \beta_0 + \beta_1\hat{q} + \frac{1}{2}V_1[y]\right). \end{aligned} \quad (\text{A.2})$$

Then, plugging the expression for  $\mathcal{E}_i[Y]$  back in the market-clearing condition gives

$$\frac{\bar{\omega} - 1}{\bar{\omega} - \underline{\omega}} = Pr\left(\exp\left((1 + \beta_1)\hat{q} + \alpha_1\nu_i + \beta_0 + \frac{1}{2}V_1[y]\right) \leq \frac{P}{1 - \omega_{\text{CEO}}} + \frac{\mathcal{E}[K^2]}{2}\right) \quad (\text{A.3})$$

Next, note that  $\sigma_x^{-1}\nu_i$  is standard normally distributed with CDF  $\Phi(\cdot)$ , which implies that

$$P = \frac{\exp\left(\alpha_1\sigma_x^{-1}\Phi^{-1}\left(\frac{\bar{\omega}-1}{\bar{\omega}-\underline{\omega}}\right) + \beta_0 + (1 + \beta_1)\hat{q} + \frac{1}{2}V_1[y]\right) - \frac{1}{2}\mathcal{E}[K^2]}{1 - \omega_{\text{CEO}}}, \quad (\text{A.4})$$

where  $V_1[y] = V_1[\theta_a] + 4\sigma_f^2 + \sigma_\varepsilon^2$  and  $\mathcal{E}[K^2] = \exp\left(2(\beta_0 + \beta_1\hat{q}) + 2(\sigma_f^2 + \sigma_\varepsilon^2)\right)$ .

As a result, the equilibrium stock price can be written as

$$P = \frac{\exp(\kappa_0 + (1 + \beta_1)\hat{q}) - \frac{1}{2}\exp(\kappa_1 + 2\beta_1\hat{q})}{1 - \omega_{\text{CEO}}}. \quad (\text{A.5})$$

It then follows that  $P$  is strictly increasing in  $\hat{q}$  if the following sufficient condition is fulfilled:

$$\alpha_1\sigma_x^{-1}\Phi^{-1}\left(\frac{\bar{\omega} - 1}{\bar{\omega} - \underline{\omega}}\right) > \frac{1}{2}\left(\frac{1}{\sigma_a^{-2} + \sigma_p^{-2}} - \frac{1}{\sigma_a^{-2} + \sigma_x^{-2} + \sigma_p^{-2}}\right) + \sigma_\varepsilon^2. \quad (\text{A.6})$$

Here the left-hand side is increasing in  $\bar{\omega}$  while the right-hand side does not depend on  $\bar{\omega}$ . Hence,  $\bar{\omega}$  has to be sufficiently large.

### A.1.2 Proof of Lemma 2

The expression for SWF follows from the fact that  $E_0[W_i]$  is proportional to  $E_0[Y - C]$ . The rest follows from plugging in the expressions for  $Y$  and  $K$  derived before.

### A.1.3 Proof of Proposition 1

If all agents become perfectly rational, the aggregate source of noise in the average expectation,  $\hat{q}$ , vanishes. As a result,  $\hat{q} = \theta_a$  and all agents can infer the realized value of  $\theta_a$  from  $\hat{q}$  or equivalently  $P$ . It then immediately follows that the CEO's optimal capital investment decision is given by  $K_{fb} = \exp(\theta_a + \theta_f)$ . Plugging  $K_{fb}$  into the expression for social welfare leads to  $SWF_{fb} = \frac{1}{2} \exp(2\sigma_a^2 + 2\sigma_f^2)$ .

### A.1.4 Proof of Corollary 1

Without information asymmetry (i.e.,  $\varepsilon = 0$ ), the CEO's optimal choice for  $K$  is identical to  $K_{fb}$  such that the resulting value for  $SWF$  is identical as well.

### A.1.5 Proof of Lemma 3

First note that the CEO's optimal capital investment (under full and near-rationality) is given by

$$k^R = \beta_0 + \beta_1 \hat{q} + \theta_f \quad (\text{A.7})$$

$$k^{NR} = \beta_0 + \beta_1 \hat{q} + \varepsilon + \theta_f, \quad (\text{A.8})$$

respectively. Then plugging  $k^R$  and  $k^{NR}$  into the CEO's expected utility gives

$$E_1[U_{\text{CEO}}^R] = \omega_{\text{CEO}} E_1 \left[ \exp(\theta_a + \theta_f + k^R) - \frac{1}{2} \exp(2k^R) \right] \quad (\text{A.9})$$

$$E_1[U_{\text{CEO}}^{NR}] = \omega_{\text{CEO}} E_1 \left[ \exp(\theta_a + \theta_f + k^{NR}) - \frac{1}{2} \exp(2k^{NR}) \right], \quad (\text{A.10})$$

where  $E_1[\cdot]$  is conditional on all information available at  $t = 1$ . Then, using the fact that  $k^{NR} = k^R + \varepsilon$  and dividing  $E_1[U_{\text{CEO}}^{NR}]$  by  $E_1[U_{\text{CEO}}^R]$  leads to the expression provided in Lemma 3.

### A.1.6 Proof of Lemma 4

The expression for  $\frac{SWF^{NR}}{SWF^R}$  simply follows from the definition of  $SWF$  together with the expressions for  $k^R$  and  $k^{NR}$ , respectively.

### A.1.7 Proof of Proposition 2

This follows from plugging in the log linear investment rule into the  $SWF$  and maximizing with respect to  $\{b_0, b_1, b_2\}$ .

### A.1.8 Proof of Lemma 5

This immediately follows from the fact that  $\alpha_1$  in the constrained efficient economy is equal to that in the benchmark equilibrium and the expression for price informativeness,  $\sigma_p^{-2} = \alpha_1^2 \sigma_\varepsilon^{-2}$ .

### A.1.9 Proof of Proposition 3

This immediately follows from the expressions for  $\{\beta_0, \beta_1\}$  in (20)-(21) and  $\{b_0, b_1\}$  in (30)-(31).

### A.1.10 Proof of Corollary 2

First note that log capital investment in the constrained first-best is given by  $k = b_0 + b_1 \hat{q} + b_2 \theta_f + \varepsilon$ . As a result, the following three moments follow:

$$E_0[k] = b_0 \tag{A.11}$$

$$V_0[k] = \left(\frac{\alpha_1 b_1}{1 - \alpha_2}\right)^2 \sigma_a^2 + \left(1 + \frac{b_1}{1 - \alpha_2}\right)^2 \sigma_\varepsilon^2 + b_2^2 \sigma_f^2 \tag{A.12}$$

$$\text{Cov}(\theta_a + \theta_f, k) = \left(\frac{\alpha_1 b_1}{1 - \alpha_2}\right) \sigma_a^2 + b_2 \sigma_f^2. \tag{A.13}$$

Next, I prove the three claims in the corollary:

- 1.) The result that  $E_0[k]$  is inefficiently high immediately follows from the fact that  $\beta_0 \geq b_0$ .
- 2.) The result that  $V_0[k]$  is inefficiently high immediately follows from the fact that  $\beta_1 \geq b_1$ .

3.) The result that  $\text{Cov}(\theta_a + \theta_f, k)$  is inefficiently high immediately follows from the fact that  $\beta_1 \geq b_1$ .

#### A.1.11 Proof of Proposition 4

This result can be easily shown by maximizing social welfare subject to  $k = b_0 + b_1\hat{q} + \beta_2\theta_f + \varepsilon$  with respect to  $\{b_0, b_1, b_2\}$  under the assumption that the CEO uses the rational expectation instead of the near-rational expectation.

#### A.1.12 Proof of Proposition 5

This follows from plugging the functional form for  $k = \gamma k_D + k_R$  into the social welfare function and maximizing with respect to  $\{\gamma, \delta_0, \delta_1\}$ .

#### A.1.13 Proof of Corollary 3

This follows from the expression for  $\gamma$  in Proposition 5. As a result,  $\frac{\partial \gamma}{\partial \sigma_f^2} = \frac{\alpha_1(\alpha_1+1)\sigma_a^2\sigma_\varepsilon^2(\alpha_1^2\sigma_a^2+\sigma_\varepsilon^2)}{(\alpha_1^2\sigma_a^2(\sigma_f^2+\sigma_\varepsilon^2)+\sigma_f^2\sigma_\varepsilon^2)^2} > 0$ .

#### A.1.14 Proof of Lemma 6

First, the set of individual state variables contains the commonly known state variables  $S_t$ . Moreover, the households build beliefs about the next period's productivity shock using their private signal and the market stock price. As a result, any individual choice  $y_i$  is a function of the state space:  $y(S_{it})$  with  $S_{it} = \{S_t, \hat{q}, \mathcal{E}_{it}[\theta_{t+1}]\}$ . Then, the equilibrium conditions result in the following form:

$$g_l(S_{it}) = \mathcal{E}_{it} [g_r(S_{it+1})]. \quad (\text{A.14})$$

Next, I show that given the structure of the right-hand side, the left-hand side is a function of the state space  $S_{it}$ . Replacing the  $g_r(\cdot)$  by its Taylor series,

$$g_r(K_{t+1}, \omega_t, \theta_{t+1}, \mathcal{E}_{CEO,t}, \hat{q}_{t+1}, \mathcal{E}_{it+1}) = \sum_j \frac{c_j(S_{it})}{j!} (K_{t+1} - K_0)^{j_1} \omega_t^{j_2} \theta_{t+1}^{j_3} \mathcal{E}_{CEO,t}^{j_4} \hat{q}_{t+1}^{j_5} \mathcal{E}_{it+1}^{j_6},$$

where  $K_0$  denotes the capital stock in the deterministic steady state,  $c_j(S_{it})$  denotes the coefficients in the Taylor series, and  $j = (j_1, j_2, j_3, j_4, j_5, j_6)$  is a multi-index for the expansion.

Next, I take the expectation conditional on  $x_{it}$  and  $\hat{q}_t$ . All terms except  $\theta_{t+1}$  and  $\mathcal{E}_{\text{CEO},t}$  are known at date  $t$  such that

$$\begin{aligned}\mathcal{E}_{it}[g_r(S_{it+1})] &= \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{c_{jk}(S_{it}, K_{t+1}, \rho\omega_{t-1} + \theta_t)}{j!k!} \mathcal{E}_{it} \left[ \mathcal{E}_{\text{CEO},t}^j \theta_{t+1}^k \right] \quad (\text{A.15}) \\ &= g_t(K_t, \omega_{t-1}, \theta_t, \mathcal{E}_{\text{CEO},t-1}, \hat{q}_t, \mathcal{E}_{it}),\end{aligned}$$

where the terms  $c_{jk}(\cdot)$  collect all the terms depending on  $K_{t+1}$  and  $\omega_t$ . Establish the second equality requires some explanation.

First, note that  $\theta_{t+1}$  and  $\mathcal{E}_{\text{CEO},t}$  are independent such that the conditional expectation can be split in two parts. Next, note that all higher moments of  $\theta_{t+1}$  and  $\mathcal{E}_{\text{CEO},t}$  are constant and known. Moreover, the conditional expectation (at date  $t$ ) of  $\mathcal{E}_{\text{CEO},t}$  is just proportional to  $\hat{q}_t$ , which implies the conditional expectation  $\mathcal{E}_{it} \left[ \mathcal{E}_{\text{CEO},t}^j \theta_{t+1}^k \right]$  is just a function of  $\hat{q}_t$  and  $\mathcal{E}_{it}$ .

Note that aggregate quantities depend on known state variables as well as the average expectation about  $\theta_{t+1}$  across households. Therefore, consider an aggregate variable:

$$Z(S_t) = \int z_i(S_{it}) di = \int \sum_j \frac{c_j}{j!} (K_t - K_0)^{j_1} \omega_{t-1}^{j_2} \theta_t^{j_3} \mathcal{E}_{\text{CEO},t-1}^{j_4} \hat{q}_t^{j_5} \mathcal{E}_{it}^{j_6} di. \quad (\text{A.16})$$

Because  $\int \mathcal{E}_{it} di = \hat{q}_t$  by definition, it holds that aggregates depend on the state of the economy  $S_t$  and the average expectation  $\hat{q}_t$ .

#### A.1.15 Proof of Lemma 7

Given Lemma 6, it follows that household  $i$ 's conditional expectation about  $\theta_{t+1}$  takes the following form:

$$\mathcal{E}_{it}[\theta_{t+1}] = \mathcal{E}[\theta_{t+1}|x_{it}, S_t] = \mathcal{E}[\theta_{t+1}|x_{it}, \hat{q}_t], \quad (\text{A.17})$$

where  $\hat{q}_t = \int_0^1 \mathcal{E}_{it}[\theta_{t+1}] di$ .

Similar to the procedure in the static model, I can then guess that  $\mathcal{E}_{it}[\theta_{t+1}] = \alpha_1 x_{it} + \alpha_2 \hat{q}_t + \varepsilon_t$ . The integrating over all households gives  $\hat{q}_t$  as a function of  $\theta_{t+1}$  and  $\varepsilon_t$ . The Bayesian weights  $\alpha_1$  and  $\alpha_2$  then simply follow from the projection theorem.

For the CEO, the conditional expectation about  $\theta_{t+1}$  is also identical to that in the static model for the same reason.

## A.2 Explicit Solution for $\alpha_1$

Here, I provide the explicit solution to the implicit expression for  $\alpha_1$  in equation (16).

Solving  $\alpha_1 = \frac{\sigma_x^{-2}}{\sigma_a^{-2} + \sigma_x^{-2} + \alpha_1^2 \sigma_\varepsilon^{-2}}$  for  $\alpha_1$  gives

$$\alpha_1 = \frac{\Phi^{1/3}}{3 \times 2^{1/3} \sigma_\varepsilon^{-2}} - \frac{2^{1/3}(\sigma_a^{-2} + \sigma_x^{-2})}{\Phi^{1/3}}, \quad (\text{A.18})$$

where  $\Phi = 27\sigma_x^{-2}\sigma_\varepsilon^{-4} + \sqrt{108(\sigma_a^{-2} + \sigma_x^{-2})^3\sigma_\varepsilon^{-6} + 729\sigma_x^{-4}\sigma_\varepsilon^{-8}}$ .

## B Appendix to Section 4

### B.1 Deriving the Equilibrium Conditions

In this section, I derive four equilibrium conditions for the quantitative model.

1. The marginal rate of substitution between consumption and labor is given by

$$\frac{1 - o}{o} \frac{C_{it}}{(1 - n_{it})} = w_t. \quad (\text{B.1})$$

2. The optimal choice of stock holdings is determined from the standard asset pricing equation:

$$\mathcal{E}_{it} [M_{it+1} R_{t+1}] = 1, \quad (\text{B.2})$$

where the stochastic discount factor is given by:

$$M_{it+1} = \delta \left( \frac{C_{it+1}}{C_{it}} \right)^{-1} \left( \frac{\tilde{C}_{it+1}}{\tilde{C}_{it}} \right)^{1 - \frac{1}{\psi}} \left( \frac{U_{it+1}}{\mathcal{E}_{it} [U_{it+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi} - \gamma}. \quad (\text{B.3})$$

3. Similarly, bond holdings are determined from

$$\mathcal{E}_{it} [M_{it+1}] (1 + r_t) - \frac{\pi'(b_{it})}{o(1 - \delta) \left(1 - \frac{1}{\psi}\right) \tilde{C}_{it}^{1 - \frac{1}{\psi}} C_{it}^{-1}} = 1. \quad (\text{B.4})$$

4. Firm investment follows from

$$\frac{1}{G'_t} = \mathcal{E}_{\text{CEO}, t-1} \left[ M_{\text{CEO}, t+1} \left( \alpha \frac{Y_{t+1}}{K_{t+1}} - \frac{I_t}{K_t} + \frac{1 - \delta + G_{t+1}}{G'_{t+1}} \right) \right], \quad (\text{B.5})$$

where  $G'_t = \frac{1}{\nu_1} \left( \frac{I_t}{K_t} \right)^{\frac{1}{\xi}}$ .



The CEO's stochastic discount factor is given by  $M_{\text{CEO},t+1} = \delta_{\text{CEO}} \left( \frac{D_{t+1}}{D_t} \right)^{-\gamma_{\text{CEO}}}$ .

## B.2 Details on the Constrained First-Best in Section 4

To compute the constrained first-best in section 4, I let the social planner choose the coefficients in the CEO's expectation of  $\theta$  to maximize ex-ante household welfare. I restrict the analysis to linear expectations of the following form:  $E_{\text{CEO},t}[\theta_{t+1}] = \delta_0 + \delta_1 \hat{q}_t + \gamma (b_0 + b_1 \hat{q}_t + \varepsilon_t + \theta_{f,t})$ . Here,  $b_0$  and  $b_1$  are the equilibrium weights chosen by the CEO. The tables in section 4 then report the combined weight on price information; that is,  $\beta_1 = \delta_1 + \gamma b_1$ . To keep the model tractable, I assume the households can write the CEO a contract enforcing the resulting investment policy. As a result, the CEO still receives the equilibrium compensation per period.

## B.3 Programming Guide

First of all, I use the following functional form for the penalty function for bond holdings<sup>29</sup>

$$\pi(b_{it}) = \frac{1}{2} \pi A_{t-1}^{1-\frac{1}{\psi}} \left( \frac{b_{it}}{A_{t-1}} \right)^2. \quad (\text{B.6})$$

Next, I define the following normalized variables:

$$\hat{X} = \frac{X}{A_{t-1}} \quad \text{for } X = C_t, K_t, I_t, C_{it}, k_{it}, b_{it}, \quad (\text{B.7})$$

and  $\hat{V}_t = \frac{V_t}{C_t}$ ,  $\hat{V}_{it} = \frac{V_{it}}{C_{it}}$ .

I can then combine the four equilibrium conditions from above together with the individual budget constraint, and the four market-clearing conditions to solve for  $\{C_t, C_{it}, n_t, n_{it}, K_{t+1}, k_{it+1}, b_{t+1}, r_t, Q_t\}$ .

I then use perturbation methods to solve the model numerically. This procedure approximates the solution to the quantitative model around the deterministic steady state. Therefore, I first solve for the deterministic steady state of the model in which all shocks' standard deviations are equal to zero. I then construct higher-order approximations for both, the deterministic and the stochastic system.

Next, I provide more details about the welfare calculations used to construct the constrained first-best in the quantitative model. Note that the share increase in lifetime

<sup>29</sup>In the numerical solution, I set this penalty equal to  $\pi = 1/1000$ .

consumption that renders a household indifferent with respect to the implementation of a given policy experiment at time 0 can be written as

$$\lambda = \frac{\log \widehat{U}_0 - \log \widetilde{U}_0}{o}, \quad (\text{B.8})$$

where  $U_0$  denotes the unconditional expected utility under the respective policy. Table 2 then displays the values for  $\lambda$ , where the two possible policies are either the benchmark equilibrium and the constrained first-best or the policy without price information and the constrained first-best.

## C Extensions of the Simple Model

### C.1 Multiple Firms

Assume the setup from section 2, with the extension that a continuum of firms indexed by  $j \in [0, 1]$  now exists. Each firm is run by an individual CEO that gets compensated in proportion to the firm's terminal payoff  $Y_j - C_j$ , where now

$$Y_j = e^{(\theta_a + \theta_j)} K_j \quad (\text{C.1})$$

$$C_j = \frac{1}{2} K_j^2. \quad (\text{C.2})$$

As before, households get a noisy private signal about  $\theta_a$ , and each CEO perfectly observes the firm-specific shock  $\theta_j$ . Here,  $\theta_j = \theta_f + e_j$ , where  $\theta_f \sim N(0, \sigma_f^2)$  and  $e_j$  is i.i.d. across firms with zero mean and variance  $\sigma_e^2$ .

As before, each CEO's optimal choice of  $k_j = \log K_j$  is given by

$$k_j = \mathcal{E}_j[\theta_a] + \frac{1}{2} V_j[\theta_a] + \theta_f + e_j, \quad (\text{C.3})$$

where  $\mathcal{E}_j[\theta_a] + \frac{1}{2} V_j[\theta_a] = \beta_0 + \beta_1 \widehat{q} + \varepsilon$  as before.

Note that because households do not have private information about (the iid) firm-specific shocks, the asset price for all firms is identical and conveys the same information about  $\theta_a$ . As a result, all agents in the economy can learn about  $\theta_a$  from observing the market price  $P = \int_0^1 P_j dj$ .

The combined welfare of all households and all CEOs is given by

$$SWF = E_0[Y - C], \quad (\text{C.4})$$

where  $Y = \int_0^1 Y_j dj$  and  $C = \int_0^1 C_j dj$ . Then, plugging in the expression for  $K_j$  from above leads to an expression for the social welfare function identical to the expression in section 3. Moreover, the individual cost for each CEO to commit  $\varepsilon$  is still equal to the expression in Lemma 3. Therefore, the two main results from the simple model carry over to this alternative setup: (i) the CEOs' private costs of near-rational investment are lower than the social costs, and (ii) they overuse price information when forming their expectations about  $\theta_a$ .